

NEWMAN'S THEOREM FOR PSEUDO-SUBMERSIONS

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1. Introduction. In 1931 M.H.A. Newman [N] proved the following result.

Theorem (Newman). *If M is a connected topological manifold with metric d , there exists a number $\varepsilon = \varepsilon(M, d) > 0$, depending only upon M and d , such that every finite group G acting effectively on M has at least one orbit of diameter at least ε .*

P.A. Smith [S] in 1941 proved a version of Newman's Theorem in terms of coverings of M and Dress [D] in 1969 gave a simplified proof of Newman's Theorem based on Newman's original approach and using a modern version of local degree.

In another direction Cernavskii [C] in 1964 generalized Newman's Theorem to the setting of finite-to-one open mappings on manifolds. His techniques were based upon those of Smith. Recently McAuley and Robinson [M-R] and Deane Montgomery [MO] have expanded upon Cernavskii's results. In fact McAuley and Robinson, using the techniques of Dress, have obtained the following version of Cernavskii's result. [M-R, Theorem 3].

Theorem (Cernavskii-McAuley-Robinson). *If M is a compact connected topological manifold with metric d , there exists a number $\varepsilon = \varepsilon(M, d) > 0$ such that if Y is a metric space and $f: M \rightarrow Y$ a continuous finite-to-one proper open surjective mapping which is not a homeomorphism, then there is at least one $y \in Y$ such that $\text{diam } f^{-1}(y) \geq \varepsilon$.*

In [H-M] we gave estimates of the ε in Newman's Theorem for Riemannian manifolds M in terms of convexity and curvature invariants of M . In this note we apply the techniques of [H-M] to obtain estimates of ε for the Cernavskii-McAuley-Robinson result for the case where M is a Riemannian manifold. In particular, if S^n is the standard unit sphere with standard metric, we show $\varepsilon > \pi/2$, i.e. if $f: S^n \rightarrow Y$ is as above, there exists $y \in Y$ with $\text{diam } f^{-1}(y) > \pi/2$. We also obtain a cohomology version of Newman's Theorem for compact orientable Riemannian manifolds which generalizes Theorem 3 of