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SOME STRUCTURE THEOREMS ON PSEUDO-SYMMETRIC SETS

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1. Introduction

In [2], a pseudo-symmetric set is defined as a binary system $(S, \circ)$ satisfying (1) $a\circ a=a$, (2) $(a\circ b)\circ c=(a\circ c)\circ(b\circ c)$ and (3) a mapping σ_a: $x\mapsto x\circ a$ is a permutation on S. The object of this paper is to develop a structuie theory for pseudo-symmetric sets. Denote σ_a by $\sigma(a)$. Then σ is considered as a mapping of S to the group of permutations on S satisfying the fundamental identity $\sigma(a^{\sigma(b)})=\sigma(b)^{-1}\sigma(a)\sigma(b)$, which results from (2). The mapping σ is called a pseudo-symmetric structure on S. The group of automorphisms generated by all σ_a is denoted by $G(S)$ or simply by G. The subgroup of G generated by all $\sigma_a^{-1}\sigma_b$ is denoted by $H(S)$. $H(S)$ is called the group of displacements of S according to the terminology in the theory of symmetric sets. It was found in previous works (See [1] and [2]) that there is a close connection between the structure of S and that of $H(S)$. In this paper, we shall investigate this connection more closely to find some structure theorems on S. To develop structure theory, we start with the concept of homomorphisms of our sets, which can be defined in a natural way. However, the concept of the kernel of a homomorphism is not available. To replace it, we introduce the concept of normal decompositions, which was already used in previous works (it was called coset-decompositions). Then, a more important concept is introduced. It is that of the group of displacements for a normal decomposition. As in the usual structure theory of groups, then, we proceed to consider sub- and factor-normal decompositions and the group of displacements for them. In the previous works, the structure of simple pseudo-symmetric sets was discussed. In this paper, we shall obtain a structure theorem on solvable (or nilpotent) pseudo-symmetric sets: S is solvable (or nilpotent) if and only if $H(S)$ is so. Lastly we remark that the theory developed here is more general. We do not need the conditions "effectiveness" and "transitivity". Also the condition (1) of the pseudo-symmetric set is not needed except for the main theorem of simple pseudo-symmetric sets. It should be also noted that the concept of normal decompositions can be applied for more general binary sys-