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CHARACTER CORRESPONDENCES IN p-SOLVABLE GROUPS

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Introduction

Let G and A be finite groups and suppose that A acts on G by automorphisms. We write $Irr(G)$ to denote the set of all irreducible characters of G over the complex number field. Then A induces permutation action on $Irr(G)$. For $\chi \in Irr(G)$ and $a \in A$, the character χ^a is defined by $\chi^a(g^a)=\chi(g)$ for $g \in G$. The set of all A-invariant characters in $Irr(G)$ is denoted by $Irr_A(G)$.

Assume further that $(|G|, |A|)=1$. G. Glauberman [2] first showed that if A is solvable then there is a bijection

$$\pi(G, A): Irr_A(G) \rightarrow Irr(C_G(A))$$

which is uniquely defined by the action of A on G.
When A is not solvable, the Odd-Order Theorem of Feit and Thompson implies that $|A|$ is even and hence $|G|$ is odd. E.C. Dade and I.M. Isaacs [3] developed the correspondence when $|G|$ is odd, and T.R. Wolf [7] showed the correspondences of Glauberman and Isaacs are equal when both are defined.

For a fixed prime p, $IBr(G)$ denotes the set of all irreducible p-modular characters of G, chosen with respect to some fixed pullback of the p-modular roots of unity to the complex numbers. Then A also induces permutation action on $IBr(G)$ by the same manner as on $Irr(G)$. Now the question arises whether there is a bijection from $IBr_A(G)$ onto $IBr(C_G(A))$ or not. The purpose of this paper is to show that it exists when G is p-solvable, namely, we shall prove the following.

Theorem. *Let A act on G such that $(|G|, |A|)=1$. Suppose that G is p-solvable. Then there exists a bijection*

$$\tilde{\pi}(G, A): IBr_A(G) \rightarrow IBr(C_G(A)).$$

And the following hold.

(i) *If $B \trianglelefteq A$, then $\tilde{\pi}(G, A)=\tilde{\pi}(G, B)\tilde{\pi}(C_G(B), A/B)$.*

(ii) *If A is a q-group for a prime q, then, for $\phi \in IBr_A(G)$, $(\phi)\tilde{\pi}(G, A)$ is the unique irreducible constituent of $\phi_{C_G(A)}$ with multiplicity prime to q.*