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ON THE INDECOMPOSABILITY OF AMALGAMATED SUMS

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Let R be a semiprimary ring with identity element. In this paper, we study when a factor module of a direct sum of local or colocal R-modules of finite length is indecomposable.

Let (E): $0\to K\xrightarrow{f}\bigoplus_{i=1}^{n}L_i\xrightarrow{g}M\to 0$ be a nonsplit exact sequence of right R-modules of finite length and f_i: $K\to L_i$ be the i^{th} coordinate map of f for each $i=1, \cdots, n$. As we will see in the paper, if M is indecomposable, then the following condition holds:

(*) For each $j=1, \cdots, n$ and each $h=(h_i)_{i=1}^{n}$: $\bigoplus_{i=1}^{n}L_i\to L_j$, $hf=\sum_{i=1}^{n}h_if_i=0$ implies that h_i is not an isomorphism for each $i=1, \cdots, n$.

The converse is not true in general, but in Tachikawa [3] we see the converse holds under rather strong conditions. Moreover in [1, section 2], we showed that this converse assertion is still true in the case of each of three groups of weaker conditions than those required in [3]. But in [1, Proposition 2.7], the third group of conditions, we assumed a condition on composition lengths of the L_i's which was not assumed in the other two cases. In this paper, we remove this condition on composition lengths and show that the condition (*) implies the indecomposability of M if each L_i is local and colocal, and each f_i is a monomorphism (see (3.3)).

In section 1, we consider the fundamental properties of the map $f=(f_i)_{i=1}^{n}$ in the sequence (E) satisfying the condition (*). Section 2 is a generalization of tools used in [1, section 2] (this generalization is not essential to understanding the main results) and in section 3 we give the main results.

Throughout the paper R is a ring with identity element, J the Jacobson radical of R, every module is a unitary right R-module. We denote by $\mathrm{Mod}\,R$ and by $\mathrm{mod}\,R$ the category of all R-modules and R-modules of finite length, respectively. We call an R-module M completely indecomposable in case the endomorphism ring $\mathrm{End}_R(M)$ is a local ring. For maps f: $K\to L$ and g: $L\to M$, and for a decomposition D: $L=\bigoplus_I L_i$ of L, the notations $(f, D)=(f_i)_I^T$ and