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Osaka J. Math.
20 (1983), 631–643

UNIPOTENT CHARACTERS OF $SO^{\pm}_{2n}$, Sp_{2n} AND SO_{2n+1} OVER F_q WITH SMALL q

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(Received April 30, 1981)

0. Introduction. Let G be a special orthogonal group or symplectic group over a finite field $\boldsymbol{F}_q$, F the Frobenius mapping and G^F the group of all F-stable points of G. G. Lusztig [7], [8] has obtained explicit formulas for the characters of the unipotent representations of G^F on any regular semisimple element of G^F provided that the order q of the defining field $\boldsymbol{F}_q$ is sufficiently large. Our purpose in this paper is to show that his formulas are valid for any q.

Let W be the Weyl group of G and m an odd positive integer. For $w \in W$, let $R_w^{(m)}$ be the Deligne-Lusztig virtual representation [2], [6, 3.4] of G^{F^m}. By [2, 7.9], to determine the values of the character of a unipotent representation ρ of G^{F^m} on regular semisimple elements, it suffices to determine the inner product

$$\langle R_w^{(m)}, \rho \rangle$$

for any $w \in W$. This has been done by G. Lusztig [7], [8] for a sufficiently large q^m. Let n be the rank of G and Ψ_n be the set of symbol classes (cf. [5, § 3]) that parameterizes the unipotent representations (up to equivalence) of G^F or G^{F^m}, i.e.

$$\Psi_n = \begin{cases} \Phi_n & \text{if } G = SO_{2n+1} \text{ or } Sp_{2n} \\ \Phi_n^{\pm} & \text{if } G = SO_{2n}^{\pm} \end{cases}$$

in the notations in [5, § 3]. For $\Lambda \in \Psi_n$, let $\rho_\Lambda^{(1)}$ and $\rho_\Lambda^{(m)}$ be the corresponding unipotent representations of G^F and G^{F^m} respectively. Our main result (Theorem 4.2, (iii)) is

$$(*) \qquad \langle R_w^{(m)}, \rho_\Lambda^{(m)} \rangle = \langle R_w^{(1)}, \rho_\Lambda^{(1)} \rangle$$

for any $\Lambda \in \Psi_n$ and $w \in W$ if m is any sufficiently large positive integer prime to $2p$ with p the characteristic of $\boldsymbol{F}_q$. Hence the required character formula is obtained for any q.

Our proof goes as follows. Firstly, we write the Frobenius mapping F