EQUIVARIANT ISOTOPIES OF SEMIFREE G-MANIFOLDS

Dedicated to Professor Minoru Nakaoka on his sixtieth birthday

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1. Introduction

In the previous paper [3] we studied the set of equivariant isotopy classes of equivariant smooth embeddings of a sphere with semifree linear action into a euclidean representation space. In this paper we will study more general case, i.e., the set of equivariant isotopy classes of equivariant smooth embeddings of a manifold into another manifold, where the manifolds in question have a smooth semifree action.

Let G be a compact Lie group, and M, N smooth G-manifolds. Two smooth G-embeddings f and g of M into N are called G-isotopic, if there is a smooth G-map

$$H: M \times [0, 1] \rightarrow N$$

such that, for any $t \in [0, 1]$, $H_t = H | M \times \{t\}$ is a smooth *G*-embedding, and that $H_0 = f$, $H_1 = g$. Such *H* is called a *smooth G*-isotopy between *f* and *g*. The *G*-isotopy class [f] is the set of all smooth *G*-embeddings *G*-isotopic to *f*. Denote by $Iso^{G}(M, N)$ the set of all *G*-isotopy classes of smooth *G*-embeddings of *M* into *N*. Fix a smooth *G*-embedding *f* of *M* into *N*, and denote by $Iso_{f}^{G}(M, N)$ the set of all *G*-isotopy classes of smooth *G*-embeddings *G*-homotopic to *f*. If *N* is a euclidean representation space of *G*, then *N* is *G*-contractible, and then

$$\operatorname{Iso}_{f}^{G}(M, N) = \operatorname{Iso}^{G}(M, N)$$

for any smooth G-embedding f of M into N.

For $x \in M$ denote by G_x the isotropy subgroup of G at x. An action of G on M is called *semifree* if, for any $x \in M$, G_x is either trivial or is all of G. If, moreover, the fixed point set

$$M^{G} = \{x \in M \mid G_{x} = G\}$$

is neither empty nor is all of M, the action is called *properly semifree*. For $x \in M^{c}$ denote by M_{x}^{c} the connected component of M^{c} containing x. Choose