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KUPKA-REEB PHENOMENA AND UNIVERSAL UNFOLDINGES OF CERTAIN FOLIATION SINGULARITIES

Dedicated to Professor Yozo Matsushima on his 60th birthday

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If $\tilde{\omega}$ is an integrable 1-form, under certain circumstances, $\tilde{\omega}$ is given as the pull back of a 1-form ω on a lower dimensional space by a submersion, that is, $\tilde{\omega}$ is a trivial unfolding of ω (Kupka-Reeb phenomenon). Especially, if we have an integrable 1-form ω which is a universal unfolding of some other 1-form, then every unfolding of ω is trivial. Thus we obtain "stable" singularities as universal unfoldings.

In this note, we construct universal unfoldings of some complex foliation singularities as an application of the versality theorem proved in [5]. For generalities on unfolding theory of complex analytic foliations, we refer to [4] and [5]. We briefly discuss universal unfoldings in section 1. In section 2, we consider the form $\omega=(\alpha x+\beta y)ydx-(\gamma x+\delta y)xdy$ on $C^2=\{(x,y)\}$ and show that, under some condition on α, β, γ and δ, we can construct a universal unfolding $\tilde{\omega}$ of ω (Theorem 2.1). As a foliation singularity, $\tilde{\omega}$ turns out to be a simple one (Remark 2.2). This fact can be used, for example, to find the solutions of the differential equation $\omega=0$ and its "perturbations". In section 3, we take up the form $\varpi=x_1\cdots x_n\sum_{i=1}^{n}a_i\frac{dx_i}{x_i}$ studied by Cerveau and Neto in [1]. They proved, among others, that every unfolding of (a form whose $n-1$ st jet is equal to) ϖ is trivial, provided that $a_i\neq a_j\neq 0$. We give (Theorem 3.2) an alternative proof of this using the versality theorem in [5]. When $n=3$ and two of the a_i's are the same, we show that some unfolding of ϖ is identical with one of the universal unfoldings constructed in section 2. We also indicate how to "stabilize" ϖ in general when two or more of the a_i's are the same (Proposition 3.8).

1. Universal unfoldings. Let $F=(\omega)$ be a codim 1 local foliation at the origin 0 in C^n ([5] section 1).

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