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ASYMPTOTIC PROPERTIES OF POSTERIOR DISTRIBUTIONS IN A TRUNCATED CASE

TADAYUKI MATSUDA

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1. Introduction

Let $X_1, \cdots, X_n$ be independent random variables with common density $f(x-\theta)$, $-\infty<x$, $\theta<\infty$, where θ is an unknown translation parameter. We shall consider here the case that $f(x)$ is a uniformly continuous density which vanishes on the interval $(-\infty, 0]$ and is positive on the interval $(0, \infty)$ and particularly

$$f(x)\sim\alpha x \qquad \text{as} \quad x\rightarrow +0$$

with $0<\alpha<\infty$.

Let $\hat{\theta}_n=\hat{\theta}_n(X_1, \cdots, X_n)$ denote the maximum likelihood estimate of θ for the sample size n. Takeuchi [4] and Woodroofe [7] showed that $\sqrt{\frac{1}{2}\alpha n\log n}\,(\hat{\theta}_n-\theta)$ has an asymptotic standard normal distribution. The speed of convergence to the standard normal distribution has been given as $O((\log n)^{s-1})$ for every fixed $s\in(0, 1)$ by the author [2] (see Theorem 1 below). Moreover, it was shown by Takeuchi [4] and Weiss and Wolfowitz [6] that $\hat{\theta}_n$ is an asymptotically efficient estimator of θ.

Woodroofe [7] also showed that if θ is regarded as a random variable with a prior density, then the posterior probability that $\sqrt{\frac{1}{2}\alpha n\log n}\,(\theta-\hat{\theta}_n)\in J$ converges to normality $\Phi\{J\}$ in probability for every finite interval J. The purpose of the present paper is to give a refinement of his result. It is shown that the variational distance between the posterior distribution and the standard normal distribution decreases of the order $(\log n)^{-s}$ with probability $1-O((\log n)^{s-1})$ for every $s\in(0, 1)$. Similar result for minimum contrast estimates in the regular case was given by Strasser [3].

2. Conditions and the main result

We shall impose the following Condition A on $f(x)$ and Condition B on a prior distribution λ.