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ON THE ISOMORPHISM THEOREM OF THE MEROMORPHIC FUNCTION FIELDS

Dedicated to Professor Kentaro Murata on his 60th birthday

TAKAO KATO

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1. Introduction

It is one of beautiful theorems in classical function theory that compact Riemann surfaces are determined by the fields of functions meromorphic on them. More precisely, let R and S be compact Riemann surfaces and let $M(R)$ and $M(S)$ be the fields of functions meromorphic on them, respectively. Then, R and S are conformally equivalent if and only if $M(R)$ and $M(S)$ are $\boldsymbol{C}$-isomorphic, i.e. there is an isomorphism of $M(R)$ onto $M(S)$ whose restriction to the field of complex numbers is the identity or the conjugate map.

For open Riemann surfaces, an isomorphism of these meromorphic function fields as abstract fields is necessarily reduced to a $\boldsymbol{C}$-isomorphism of them (Iss'sa [2]). In spite of this fact, there is a pair of compact Riemann surfaces R and S of genus g ($\geqq 1$) which are not conformally equivalent and $M(R)$ and $M(S)$ are isomorphic as abstract fields. It was noted by Heins [1] for $g=1$ and the author [3] for $g\geqq 2$. Nakai and Sario [5], however, showed that if $M(R)$ and $M(S)$ are isomorphic, then R and S are topologically equivalent.

Let I_g be the set of compact Riemann surfaces of genus g satisfying the following condition: R is an element of I_g if for any Riemann surface S such that $M(R)$ is isomorphic to $M(R)$, S and R are conformally equivalent. In this paper we shall study how many elements of I_g do there exist in H_g the space of hyperelliptic Riemann surfaces of genus g.

2. Statement of Theorem

For the sake of simplicity, henceforce, we restrict the meaning of the terms *isomorphism* and *conformal* to the following: *isomorphisms* of fields we mean are *direct* isomorphisms, i.e. $\sqrt{-1}$ is mapped to $\sqrt{-1}$, and *conformal* does not mean *indirect* (or *anti-*) conformal.

Let $a_1, \cdots, a_{2g-1}$ ($g\geqq 2$) be mutually distinct complex numbers. Then, there is a hyperelliptic Riemann surface of genus g defined by the equation