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ON A CONSTRUCTION OF NULL ELECTROMAGNETIC FIELDS

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Introduction. In this paper we consider Maxwell's equations with a certain nonlinear condition and give an elementary method of constructing the solutions of these.

After the work of Penrose [5], complex manifold techniques have been used for representing the solutions of Maxwell's equations. It is now known that the solutions are represented in terms of cohomology classes on an open complex manifold with coefficients in a certain holomorphic vector bundles (cf. Penrose [5], Wells [7]). But it is not always easy to have the solutions in the explicit form using this representation. The purpose of this paper is to give a direct method of constructing the solutions. Our approach is based on the work of Robinson [6]. In [6] he studied a particular class of the solutions, so-called null electromagnetic fields and found the connection between these fields and the special families of null lines.

We give a brief summary of the results of [6]. The solutions of Maxwell's equations, namely, electromagnetic fields are represented by means of the differential 2-forms on Minkowski space. The differential 2-forms induce the linear mappings from the tangent space to the contangent space by contraction. The intersection N of the kernels of the transformations induced by F and $*F$ plays an essential role, where F is a differential 2-form and $*F$ is the Hodge dual of F. If F is a null electromagnetic field, N has dimension 1 and is null. Therefore we have a family of null lines (null rays associated with a null electromagnetic field). This family satisfies some nonlinear equations which are called shear-free equations. We say that a family of null lines is a shear-free null congruence if it satisfies shear-free equations. Null electromagnetic fields are constructed from shear-free null congruences.

In the process of carrying out Robinson's method of constructing null electromagnetic fields we must solve an overdetermined system of differential equations (3.11) which has coefficients related to a shear-free null congruence. In the present paper we solve Eqs. (3.11) exactly and construct null electromagnetic fields. At this stage the theorem of Kerr which asserts that every analytic shear-free null congruence is obtained from a complex analytic homo-