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Osaka J. Math.
20 (1983), 217-236

ANALYTICITY OF SOLUTIONS OF QUASILINEAR EVOLUTION EQUATIONS II

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(Received May 15, 1981)

0. Introduction

In this paper we establish analyticity in t of solutions to quasilinear evolution equations

$$(0.1)\quad \frac{du}{dt}+A(t, u)u = f(t, u), \qquad 0\leqq t\leqq T,$$

$$(0.2)\quad u(0) = u_0.$$

The unknown, u, is a function of t with values in a Banach space X. For fixed t and $v\in X$, the linear operator $-A(t, v)$ is the generator of an analytic semigroup in X and $f(t, v)\in X$. In the case that the domain $D(A(t, v))$ of $A(t, v)$ does not depend on t and v, Massey [7] discussed analyticity in t for equation of the form (0.1).

In the present paper, we consider analyticity for (0.1), (0.2) under the assumptions that the domain $D(A(t, v)^h)$ of $A(t, v)^h$ is independent of t, v for some $h=1/m$ where m is a positive integer and that $A(t, A_0^{-\alpha}v)^h$ is Hölder continuous in v in the sense that $\|A(t, A_0^{-\alpha}v)^h A(t, A_0^{-\alpha}w)^{-h}-I\|\leqq C\|v-w\|^\eta$, while in the previous papers [2], [3] we discussed the same problem in the case that $A(t, A_0^{-\alpha}v)^h$ is Lipschtz continuous. In order to prove the theorems we shall make use of the linear theory of Kato [5].

In the following $L(X, Y)$ is the space of linear operators from a normed space X to another normed space Y, and $B(X, Y)$ is the space of bounded linear operators belonging to $L(X, Y)$. $L(X)=L(X, X)$ and $B(X)=B(X, X)$. $\|\ \|$ will be used for the norm in both X and $B(X)$; it should be clear from the context which is intended. $\sum(\phi; T)\equiv\{t\in \boldsymbol{C};\ |\arg t|<\phi, 0<|t|<T\}\cup\{0\}$ is the sector in the complex plane.

We shall make the following assumptions:
(A-1) There exist $h=1/m$, where m is an integer, $m\geqq 2$, and $0\leqq\alpha<h/2$ such that $A_0^{-\alpha}$ is a well-defined operator $\in B(X)$ and $u_0\in D(A_0^{1+\alpha})$ where $A_0\equiv A(0, u_0)$.
(A-2) A_0^{-1} is a completely continuous operator.