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ISOPARAMETRIC TRIPLE SYSTEMS OF ALGEBRA TYPE

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Introduction. In this paper we continue our study of isoparametric triple systems. These triple systems have been introduced in [3] and are studied in [3], [4] and [5]. They are in one-to-one correspondence with isoparametric hypersurfaces in spheres which have four distinct principal curvatures.

The classes of isoparametric hypersurfaces which have been considered up to now are the homogeneous ones ([10], [11]), the surfaces of FKM-type ([5], [6]), the surfaces satisfying "condition (A) and (B)" ([9], [10]) and the surfaces where the multiplicity of one of the principal curvatures is ≤ 2 ([12], [10]).

However, until now there exists no classification of all isoparametric hypersurfaces in spheres. It therefore may be useful to investigate special types of hypersurfaces, i.e., special types of isoparametric triple systems. In this paper we classify isoparametric triples of algebra type. Such triples correspond uniquely to those isoparametric hypersurfaces which satisfy the "condition (A)" of [9], but not necessarily the additional "condition (B)" of [9].

The classification is summarized in Theorem 5.18. As a corollary we get that every isoparametric triple of algebra type is equivalent to a hypersurface of FKM-type or to one 8-dimensional homogeneous hypersurface.

The paper is organized as follows: In section 1 we introduce the basic notations and mention some fundamental results concerning isoparametric triple systems. Next, we reduce the problem of describing isoparametric triples of algebra type to the problem of classifying certain families of representations of Clifford algebras. The result indicates that one has to consider the cases $m_1>m_2+1$, $m_1=m_2+1$ and $m_1=m_2$ separately (where m_1 and m_2 are the multiplities of the principal curvatures). This is done in the next 3 sections. In each case we explicitly determine the isomorphism classes of the corresponding triple systems. As an application of our results we show in the last section that every isoparametric triple system which is 'generically' of algebra type is already homogeneous.

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