Tanaka, R.
Osaka J. Math.
20 (1983), 137-143

ON THE STABLE JAMES NUMBERS OF THOM COMPLEXES

RYUICHI TANAKA

(Received May 12, 1981)

1. Introduction

Let X be a connected finite CW-complex with a base point, and ξ be a (real) vector bundle over X. We have the natural inclusion map

$$i: S^n = (pt)^n \to X^\xi$$

of Thom complexes, where $n=\dim \xi$ and the trivial n-dimensional bundle is also denoted by n for brevity. Consider the homomorphism

$$i^*: \{X^\xi, S^n\} \to \{S^n, S^n\} = Z$$

of stable cohomotopy groups. Then the stable James number of X^ξ, which we shall denote by $d(X, \xi)$, is defined to be the non-negative generator of image i^* (see [7]). Thus $d(X, \xi)$ is the least positive integer r such that a map $S^n \to S^n$ of degree r can be stably extended to X^ξ, if it exists, or zero otherwise. For a map $f: X^\xi \to S^n$, we shall call the degree of $f\circ i$ the degree of f simply.

Suppose, for example, that X is the projective space FP^{k-1} ($F=C$ or H), and ξ is n-fold Whitney sum of the canonical line bundle η over FP^{k-1}, then $X^\xi = FP^{k+n-1}/FP^{n-1}$ and $d(FP^{k-1}, n\eta)$ is the same as $F\{n, k\}$ in [9]. In that paper, Ōshima determined $F\{n, k\}$ for several small k's (see also [3], [7] and [8] for $F\{1, k\}$).

Now let X and ξ be as before. Let $J(X)$ denote the group of stable fibre homotopy equivalence classes of real vector bundles over X, and $J(\xi)$ the class of ξ in $J(X)$. Since a stable fibre homotopy equivalence of bundles induces a stable homotopy equivalence of their Thom complexes, we may regard $d(X, -)$ as a function from $J(X)$ to Z. We shall abuse notations, and not distinguish $d(X, J(\xi))$ from $d(X, \xi)$. Our main result is as follows:

Theorem 1.1. *Let p be a prime number, and suppose that ξ is an orientable vector bundle over X. Then,*

(1) *$d(X, \xi)$ is not zero*

(2) *p is a divisor of $d(X, \xi)$ if and only if p is a divisor of the order of $J(\xi)$.*