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# *BP* OPERATIONS AND HOMOLOGICAL PROPERTIES OF $BP_*BP$-COMODULES

ZEN-ICHI YOSIMURA

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$BP$ is the Brown-Peterson spectrum for a fixed prime $p$ and $BP_*X$ is the Brown-Peterson homology of the $CW$-spectrum $X$. The left $BP_*$-module $BP_*X$ is an associative comodule over the coalgebra $BP_*BP$. In [2] we have studied some torsion properties of (associative) $BP_*BP$-comodules, by paying attension to the behaviors of $BP$ operations. It seems that the following result is fundamental.

**Theorem 0.1.** *Let $M$ be a $BP_*BP$-comodule. If an element $x \in M$ is $v_n$-torsion, then it is $v_{n-1}$-torsion.* ([2, Theorem 0.1]).

After a little while Landweber [8] has obtained several results about torsion properties of associative $BP_*BP$-comodules in an awfully algebraic manner, as new applications of commutative algebra to the Brown-Peterson homology. In this note we will give directly new proofs of Landweber's principal results [8, Theorems 1 and 2], by making use of two basic tools (Lemmas 1.1 and 1.2) looked upon as generalizations of Johnson-Wilson results [1, Lemmas 1.7 and 1.9] handling BP operations:

**Theorem 0.2.** *Let $M$ be a $BP_*BP$-comodule and $x \neq 0$ be an element of $M$. Then the radical of the annihilator ideal of $x$*

$$\sqrt{\mathrm{Ann}(x)} = \{\lambda \in BP_*;\ \lambda^k x = 0 \text{ for some } k>0\}$$

*is one of the invariant prime ideals $I_n=(p, v_1, \cdots, v_{n-1})$ in $BP_*$, $1 \leqq n \leqq \infty$.* (Theorem 1.3).

**Theorem 0.3.** *Let $M$ be an associative $BP_*BP$-comodule and $1 \leqq n < \infty$. If $M$ contains an element $x$ satisfying $\sqrt{\mathrm{Ann}(x)} = I_n$, then there is a primitive element $y$ in $M$ such that the annihilator ideal of $y$*

$$\mathrm{Ann}(y) = \{\lambda \in BP_*;\ \lambda y = 0\}$$

*is just $I_n$.* (Theorem 2.2).