Minami, H.
Osaka J. Math.
20 (1983), 109-122

ON EQUIVARIANT J-HOMOMORPHISM FOR INVOLUTIONS

HARUO MINAMI

(Received April 17, 1981)

Introduction. Let G be the cyclic group of order 2.

We denote by $\pi_S^{*,*}$ the equivariant stable cohomotopy theory [2, 3] and by KO_G^* the K-theory of real G-vector bundles on G-spaces. For a finite pointed G-complex we then have an equivariant J_G-map $J\colon \widetilde{KO}_G^{-1}(X)\rightarrow \pi_S^{0,0}(X)$ [14], which becomes a homomorphism if X is a suspension in the usual sense.

Let $R^{p,q}$ be the euclidean space R^{p+q} with non trivial G-action on the first p coordinates and $\Sigma^{p,q}$ be the one point compactification of $R^{p,q}$, with ∞ as base point. We have the canonical isomorphism $\pi_S^{0,0}(\Sigma^{p,q})\approx\pi_{p,q}^S$, the (p, q)-th equivariant stable homotopy group of Landweber [9, 3] (which is $\pi_{p+q,p}$ of Bredon [5]), and therefore we get an induced map

$$\widetilde{KO}_G^{-1}(\Sigma^{p,q})\rightarrow \pi_S^{0,0}(\Sigma^{p,q})\approx\pi_{p,q}^S$$

which we also denote by J_G. P. Löffler [10] showed that if $\widetilde{KO}_G^{-1}(\Sigma^{p,q})$ is a torsion group then J_G is a split injection. In this paper we shall study the image of J_G when $\widetilde{KO}_G^{-1}(\Sigma^{p,q})$ is torsion free. And then we shall give a supplement to [11] on $\mathrm{Im}\, J_R$. The J_G is also studied by M.C. Crabb [6].

We denote by Z/n a cyclic group of order n, by $R\cdot x$ the free module over a ring R generated by x. If $p\equiv i \bmod 8$ and $q\equiv j \bmod 8$, then we write $(p, q)\equiv (i, j) \bmod 8$.

The author would like to express his gratitude to Professor S. Araki for his kindly advice.

1. The J-homomorphism J_G

In this section we shall give the relations between various homomorphisms and collect some basic tools. Let X be a finite pointed G-complex.

Let KR denote the K-functor of [4]. By regarding a Real vector bundle on X as a real G-vector bundle on X we get a homomorphism $\sigma\colon \widetilde{KR}^{-1}(X)\rightarrow \widetilde{KO}_G^{-1}(X)$. We define a map $J_R\colon \widetilde{KR}^{-1}(X)\rightarrow \pi_S^{0,0}(X)$ by

$$J_R = J_G\sigma \tag{1.1}$$