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ON A GENERALIZATION OF SEMIPERFECT MODULES

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In this paper we shall generalize the notion of semiperfect modules in terms of preradicals, and show that almost all properties of semiperfect modules are preserved under this generalization. In particular we can derive immediately new characterizations of semiperfect rings and modules (Corollaries 1.4 and 1.8 below). Namely, when one deals with perfect or semiperfect rings, one can drop the smallness assumption from the definition of a projective cover.

Throughout this paper R will always denote a ring with identity and all modules will be assumed to be unital right R-modules, unless otherwise specified. For any module M we shall denote its Jacobson radical by $J(M)$. A submodule N of M is said to be *small* in M if $T+N=M$ implies $T=M$ for any submodule T of M. Dually, N is said to be *large* in M if $T\cap N=0$ implies $T=0$. By a *preradical* we shall mean a subfunctor of the identity functor on the category of modules. We refer to [5] for details concerning preradicals. Also we shall use freely the definitions and results of Bass [1] and Mares [2] in what follows.

1. A generalization of semiperfect modules

We start with some definitions: Let M be any module and ρ any preradical on modules. Then we shall say that an epimorphism $P\xrightarrow{\pi}M\to 0$ is a *ρ-semicover* of M if P is a projective module and if $\operatorname{Ker}\pi\subset\rho(P)$. A projective module will be said to be *ρ-semiperfect* (resp. *ρ-perfect*) if any factor module of it (resp. of any direct sum of its copies) has a ρ-semicover. If a ring R is ρ-semiperfect as a right module, then we call R a *ρ-semiperfect ring*. Similarly we define a *ρ-perfect ring*.

Clearly a projective module is ρ-perfect if and only if any direct sum of its copies is ρ-semiperfect. As we see later (Theorem 1.3 below), it turns out that ρ-semiperfect and semiperfect modules are closely related to each other.

Basic facts about semicovers which will be needed later are summarized in the following lemmas. Before proving those we note the useful fact that $\rho(P)=P\rho(R)$ for any preradical ρ and any projective module P.