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LOCAL PROPERTIES OF p-BLOCK ALGEBRAS OF FINITE GROUPS

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I. Introduction

The Brauer correspondence is one of the basic methods in the theory of modular representations of finite groups. Moreover, some of the other 1–1 correspondences (of modules or blocks) share good relationships with it. For example, the Green correspondence is one of such ones (Nagao, see (7.7) Feit [8]) and it was noted by Alperin [1] that the Glauberman correspondences of characters for relatively prime operator groups may be proved via the First Main Theorem of Brauer. Here, we pick up what we call the Fong correspondences of blocks and study the relationships of them with the Brauer one. In particular, we get a generalization of one of the results in Okuyama and Wajima [12].

Our second concern is with a certain special p-block algebra of a finite group —one which is separable over its center. Such a block has an extreme property on the number of its characters, namely if d is the defect of it, then it has exactly p^d ordinary irreducible characters and one modular irreducible character. According to Brauer [4], this happens if the block has the inertial index one and its defect group is abelian. We shall show that the converse of this fact is true, so that the separability is completely characterized by the local property (except the case of defect zero, of course). The proof will be carried out through the analysis on the annihilator ideal of the radical of a group algebra, especially of its center, which originates in a Brauer's old remark [3] back in 1950's. Some general facts about separable algebras are also helpful.

The notation is standard. G will always denote a finite group and p a prime number. We fix a complete, discrete rank one valuation ring R, with quotient field L of characteristic zero and residue field k of characteristic p. We assume that L has the $|G|$-th roots of unity. If a is an element of an R-module M, then $\bar{a}$ denotes its image under the natural map $M \to k \otimes M = \bar{M}$, where $\otimes = \otimes_R$ (throughout this paper). By a p-block of G, we mean here a block (ideal) of the group ring $R[G]$. If A is a ring, then $J(A)$ and $Z(A)$ denote its Jacobson radical and its center respectively and for a subset S of A, $(0: S)$ denotes the set of right annihilators of S in A. Finally, $M(n, A)$ denotes the full matrix