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ON THE ZETA FUNCTIONS OF THE VARIETIES $X(w)$ OF THE SPLIT CLASSICAL GROUPS AND THE UNITARY GROUPS

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0. Introduction

Let G be one of the split classical groups SO_{2n}^{+}, SO_{2n+1}, Sp_{2n} or a unitary group defined over the finite field $\boldsymbol{F}_q$ of q elements. Let F be the Frobenius mapping, G^F the subgroup of F-stable elements, W the Weyl group of G and let δ be the smallest positive integer such that F^{δ} acts trivially on W. For $w\in W$, Deligne-Lusztig [3] has defined the F^{δ}-stable variety $X(w)$ for any connected reductive group. If w is a Coxeter element of W, the zeta function of $X(w)$ was obtained by Lusztig [9] as a by-product when he determined the Green polynomial associated with w. In this paper we shall determine the zeta function of $X(w)$ for any $w\in W$.

To state our result more explicitly, let B be a fixed F-stable Borel subgroup of G, $\mathfrak{A}^K(W)$ the Hecke algebra of the representation of G^{F^m} induced from the trivial representation of B^{F^m} and let $\{a_w^K;\ w\in W\}$ be the natural basis of $\mathfrak{A}^K(W)$. When δ divides m the number of F^m-stable points of $X(w)$ is expressed in terms of the dimensions of the unipotent representations of G^F and the trace of a_w^K on each irreducible representation of $\mathfrak{A}^K(W)$.

The crucial point of our arguments depends on the lifting theory due to Shintani-Kawanaka ([15], [7], [8]) and a result of Lusztig ([12], Corollary 3.9), which says that for any unipotent representation ρ of G^F, the eigenvalues of F^{δ} on the ρ-isotypic component of $H_c^i(X(w))$ are independent of i and w up to a multiple factor of the form $q^{i\delta}$, $i\in\boldsymbol{Z}$.

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1. General results

1.1. First we summarize the known results (Shintani [14], Kawanaka [7], [8]) to apply for our use.

Let m be a positive integer (maybe 1), $k=\boldsymbol{F}_q$, $K=\boldsymbol{F}_{q^m}$, G a connected algebraic