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A PAIR OF SUBALGEBRAS IN AN AZUMAYA ALGEBRA

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Introduction

It was shown by A.A. Albert [1] that a cyclic central simple (Azumaya) p-algebra has a purely inseparable extension field as a subalgebra. Hence such algebra contains a cyclic extension and a purely inseparable extension as subalgebras. On the other hand, a quaternion algebra $\boldsymbol{R}(i, j)$ contains two cyclic extensions $\boldsymbol{R}(i)$ and $\boldsymbol{R}(j)$ as subalgebras. These subalgebras are related by inner actions.

In this paper, we shall generalize the above results using Hopf algebras since we must treat separable and inseparable extensions simultaneously. Under certain conditions we shall show that if an Azumaya algebra A over a field K contains an H-Hopf Galois extension (in the sense of [3], [15]) of K as a maximal commutative subalgebra then A contains an H^*-Hopf Galois extension of K as a subalgebra. This is done in §1. In §2, we shall treat typical Hopf algebras and show that the classical results are typical examples of our results.

Throughout this paper, K will denote a field and H will denote a finite commutative co-commutative Hopf algebra over K. ε (resp. Δ, resp. λ) will denote augmentation (resp. diagonalization, resp. antipode) of H. Unadorned $\otimes$ and Hom will mean $\otimes_K$ and Hom_K. We shall denote by $-^*$ the functor $\mathrm{Hom}_K(-, K)$. For Hopf algebras and Hopf Galois extensions we shall refer to [3], [10], [15] and [16].

1. Hopf Galois extension $v(H)$

Let A be a K-Azumaya algebra which contains an H-Hopf Galois extension L of K as a maximal commutative subalgebra. Then the H-action on L is extended innerly to the action on A and A is a smash product algebra $L\,\sharp_\sigma H$ (c.f. [14] cor. 3.7 and 3.8). But in general A would not become an H-module if we extend the H-action on L innerly to A. The following Proposition is fundamental.

Proposition 1. *The following conditions are equivalent.*
(i) *A becomes an H-module algebra.*