Morimoto, M.
Osaka J. Math.
19 (1982), 745-761

A SPLITTING PROPERTY OF ORIENTED HOMOTOPY EQUIVALENCE FOR A HYPERELEMENTARY GROUP

MASAHARU MORIMOTO

(Received December 5, 1980)

1. Introduction

Let G be a finite group. In this paper a G-space means a complex G-representation space of finite dimension. For a G-space V we denote by $S(V)$ its unit sphere with respect to some G-invariant inner product. After tom Dieck [1] and [2] we call two G-spaces V and W oriented homotopy equivalent if there exists a G-map f: $S(V)\rightarrow S(W)$ such that for each subgroup H of G the induced map $f^H:S(V)^H\rightarrow S(W)^H$ on the H-fixed point sets has degree one with respect to the coherent orientations which are inherited from the complex structures on V^H and W^H. Let $R(G)$ be the complex G-representation ring, $R_h(G)$ the additive subgroup of $R(G)$ consisting of $x=V-W$ such that V and W are oriented homotopy equivalent, and $R_0(G)$ the additive subgroup of $R(G)$ consisting of $x=V-W$ such that dim $V^H=$dim W^H for all the subgroups H of G. We denote by $j(G)$ the quotient group $R_0(G)/R_h(G)$.

If G has a normal cyclic subgroup A and a Sylow p-subgroup H such that G is the semidirect product of H by A, we call G a hyperelementary group. Especially if G is the direct product of A and H, we call G an elementary group. tom Dieck showed that for an arbitrary finite group G the restriction homomorhpism from $j(G)$ to the direct sum of $j(K)$ is injective, where K runs over the hyperelementary subgroups of G ([1; Proposition 5.1]). Our purpose of this paper is to consider oriented homotopy equivalence for hyperelementary groups and to give a sufficient condition for a hyperelementary group to have a splitting property defined below.

Choose an integer m which is a multiple of the orders of the elements of G, and let $Q(m)$ be the field obtained by adjoining the *m-th* roots of unity to Q, where Q is the field of rational numbers. The Galois group $\Gamma=\Gamma(m)$ of $Q(m)$ over Q acts on $R(G)$ via its action on character value. Actually Γ acts on the set $Irr(G)$ of isomorphism classes of irreducible G-spaces. Let $Z[\Gamma]$ be the integral group ring of Γ, and $I(\Gamma)$ its augmentation ideal. Then we have $R_0(G)=I(\Gamma)R(G)$. We put $R_1(G)=I(\Gamma)R_0(G)$. According to [3] we have $R_1(G)\subset R_h(G)$. Let us say that G has *Property* 1 if $R_1(G)$ coincides with $R_h(G)$.