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TOTALLY REAL SUBMANIFOLDS AND SYMMETRIC BOUNDED DOMAINS

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Introduction. Let $P_n(c)$ denote the complex projective n-space endowed with the Kählerian metric of constant holomorphic sectional curvature $c>0$. We consider an n-dimensional complete totally real submanifold M of $P_n(c)$ with parallel second fundamental form σ. The first named author [6] reduced the classification of such submanifolds to that of certain cubic forms of n-variables, and he classified completely those without Euclidean factor among such submanifolds. (Note that such a submanifold is always locally symmetric.)

In this note we shall give another way of the classification of these submanifolds. Let $D\subset \boldsymbol{C}^{n+1}$ be a symmetric bounded domain of tube type realized by the Harish-Chandra imbedding. We imbed the Shilov boundary $\hat{M}$ of D into the hypersphere $S^{2n+1}(c/4)$ of the radius $2/\sqrt{c}$ with respect to a suitable hermitian inner product of $\boldsymbol{C}^{n+1}$. Let $M\subset P_n(c)$ be the image of $\hat{M}$ under the Hopf fibering $\pi\colon S^{2n+1}(c/4)\rightarrow P_n(c)$. Then M is an n-dimensional complete totally real submanifold with parallel second fundamental form (Theorem 2.1), and conversely such a submanifold is obtained in this way (Theorem 3.1). The crucial point in the argument is that $M\subset P_n(c)$ has the parallel second fundamental form if and only if $\hat{M}=\pi^{-1}(M)\subset S^{2n+1}(c/4)$ has the parallel second fundamental form (Lemma 1.1). Thus we may use the classification (Ferus [3], Takeuchi [10]) of submanifolds in spheres with parallel second fundamental form.

As an application, we give a characterization of an n-dimensional compact totally real minimal submanifold M of $P_n(c)$ with $||\sigma||^2=n(n+1)c/4(2n-1)$. (Recall that $||\sigma||^2<n(n+1)c/4(2n-1)$ implies $\sigma=0$. cf. Chen-Ogiue [1].) Such a submanifold M is unique and nothing but the flat isotropic surface $M_0^2\subset P_2(c)$ with parallel second fundamental form constructed in Naitoh [5] (Theorem 4.5).

1. Hopf fiberings

Let $\boldsymbol{R}^{n+1}$ be the real Cartesian $(n+1)$-space with the standard inner pro-

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