Kawakubo, K.
Osaka J. Math.
19 (1982), 695–715

ΛG-STRUCTURE OF G-VECTOR BUNDLES AND GROUPS $KO_G(X)$, $KSp_G(X)$ and $J_G(X)$

KATSUO KAWAKUBO

(Received September 18, 1978)
(Revised January 12, 1982)

1. Introduction

Let G be a compact topological group. We say that X is a trivial G-space if X is a topological space with the G-action $gx=x$ for all $g\in G$ and all $x\in X$. Let V_i run over the inequivalent irreducible complex G-representations. For any complex G-representation V, there is a canonical isomorphism

$$(*)\qquad \bigoplus_i V_i \underset{G}{\otimes} \mathrm{Hom}_{CG}(V_i, V) \cong V\,.$$

Using this isomorphism, Atiyah and Segal had a decomposition of a complex G-vector bundle over a compact trivial G-space X [4]. As a consequence they showed that the equivariant complex K-group $K_G(X)$ is isomorphic to the tensor product $R(G)\otimes K(X)$ of the complex representation ring $R(G)$ and the complex K-group $K(X)$.

In the present paper, we first make real and symplectic versions of these for our later use, although they seem familiar to us all (Propositions 3.1 and 4.1).

Similar decompositions have been already obtained for some special cases; by Conner-Floyd [7] for G a cyclic group of odd prime order, by Atiyah-Singer [5] for G a monogenic group, and by Uchida [25] for semi-free S^1-and S^3-actions.

Moreover we show that the decompositions of real and symplectic G-vector bundles are unique up to isomorphism in respective category (Proposition 4.2).

As an application, we express the equivariant real K-group $KO_G(X)$ and the equivariant quaternionic K-group $KSp_G(X)$ in terms of irreducible G-representations and their types, the real K-group $KO(X)$, the complex K-group $K(X)$ and the quaternionic K-group $KSp(X)$ (Theorems 5.1 and 5.2) (Compare [21] for $KO_G(X)$). Consequently we know that the real version of the Atiyah-Segal theorem above does not hold in a similar form in general.

Research supported in part by the Fujukai Foundation and Grant-in-Aid for Scientific Research.