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LÉVY'S FUNCTIONAL ANALYSIS IN TERMS OF AN INFINITE DIMENSIONAL BROWNIAN MOTION I

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0. Introduction.

In the "Troisième Partie: La notion de moyenne dans l'espace fonctionnel et l'équation de Laplace généralisée" of his book "Problèmes concrets d'analyse fonctionnelle", Paul Lévy has extensively developed a potential theory on an infinite dimensional space. About this work he says in his autobiography (Lévy [2], p. 63): "Aussi la troisième partie de mon lirve est-elle une esquisse à grands traits.".

The purpose of this paper is to give a rigorous formulation of some aspects of "une esquisse à grands traits". That is, we shall construct an infinite dimensional space $\boldsymbol{E}$ and an infinite dimensional Brownian motion $\boldsymbol{B}$ with state space $\boldsymbol{E}$. Then we shall describe some results of his potential theory in terms of the infinite dimensional Brownian motion $\boldsymbol{B}$.

We shall now explain the noticeably different features of Lévy's theory from the ones of recent works (e.g., Daletskii [3], Gross [8], Hida [11]). In his potential theory, such objects as his infinite dimensional Laplacian and harmonic functions are regarded as limits of sequences of the corresponding ones in the finite dimensional Euclidean spaces R^N, as $N\rightarrow\infty$. He calls this finite dimensional construction method "la méthode du passage du fini à l'infini". In the potential theory on the space R^N, the volume element, surface elements, the Laplacian Δ_N, Poisson kernels on the balls and so forth are determined in terms of the Riemannian metric $ds_N^2=dx_1^2+\cdots+dx_N^2$. As is seen in Green-Stokes' formula, these objects are compatible with each other. Hence the mutual compatibility could be inherited through his finite dimensional constiuction method by the various objects in Lévy's potential theory. In particular, the coordinates in his infinite dimensional space could be regarded as equally weighted. On the other hand, in Lévy's theory harmonic functions can be discontinuous (see Lévy [1], pp. 305–306). Hence the Laplacian as a differential operator would be rather inconvenient in treating such pathological phenomena.

Now we shall reformulate Lévy's potential theory according to his finite dimensional construction method. Motivated by Gâteaux and Lévy's derivation