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HOLOMORPHIC ISOMETRIC IMBEDDING INTO $Q_m(C)$

YOSHIHIKO SUYAMA

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0. Introduction

In this paper, we shall study the existence and rigidity problems of a holomorphic isometric imbedding of a Kaehler manifold into a complex quadric $\boldsymbol{Q}_m(\boldsymbol{C})$.

A systematic study of the holomorphic isometric imbeddings of Kaehler manifolds with analytic metrics was done by E. Calabi [2]. He considered the so-called diastatic function of a Kaehler manifold and showed that this function plays an important role in study of the holomorphic isometric imbedding [see §1 in this paper]. Especially giving an explicit representation of the diastatic function of a simply connected complete Kaehler manifold with constant holomorphic sectional curvature, he found a necessary and sufficient condition on a Kaehler manifold $\boldsymbol{M}$ in order that a holomorphic isometric imbedding of $\boldsymbol{M}$ into this space exists. And then he proved the rigidity theorem for such an imbedding.

We shall give here an explicit representation of the diastatic function of $\boldsymbol{Q}_m(\boldsymbol{C})$. By making use of this function, we shall make a special coordinate system in $\boldsymbol{Q}_m(\boldsymbol{C})$ around each point [§2]. These representation and coordinate system are the core of this paper [§3 and §4].

The complex quadric $\boldsymbol{Q}_m(\boldsymbol{C})$ is a complex hypersurface in the projective space $\boldsymbol{P}_{m+1}(\boldsymbol{C})$ defined by

$$(z^0)^2+(z^1)^2+\cdots+(z^{m+1})^2=0$$

with respect to the homogeneous coordinate system $(z^0, \cdots, z^{m+1})$ of $\boldsymbol{P}_{m+1}(\boldsymbol{C})$. As a Kaehler metric on $\boldsymbol{Q}_m(\boldsymbol{C})$, we take the metric induced from that on $\boldsymbol{P}_{m+1}(\boldsymbol{C})$, which is the Fubini-Study metric with constant holomorphic sectional curvature 4. The complex quadric $\boldsymbol{Q}_m(\boldsymbol{C})$ has the group of holomorphic isometric transformations, which acts on $\boldsymbol{Q}_m(\boldsymbol{C})$ transitively.

We shall show that $\boldsymbol{P}_n(\boldsymbol{C})$ is holomorphically and isometrically imbedded into $\boldsymbol{Q}_l(\boldsymbol{C})$ only for $l\geqq 2n$ [§3, Ex. 3 and Th. 1]. This implies that, for a Kaehler manifold $\boldsymbol{M}$, the existence problem of a holomorphic isometric imbedding of $\boldsymbol{M}$ into $\boldsymbol{Q}_m(\boldsymbol{C})$ is equivalent to that of such an imbedding into $\boldsymbol{P}_n(\boldsymbol{C})$.

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