Gordon, C.
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SEMISIMPLE NORMAL SUBGROUPS OF TRANSITIVE RIEMANNIAN ISOMETRY GROUPS

CAROLYN GORDON[1]

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1. Introduction. In this paper we prove the following:

Theorem. *Suppose the connected Lie group A is a product $A=GL$ of a connected subgroup G and a compact subgroup L. Let H be a connected semisimple normal subgroup of G. Then*
(a) *if H is of noncompact type, H is normal in A;*
(b) *if H is compact, then H is contained in a compact semisimple normal subgroup of A.*

Here H "of noncompact type" means all simple connected normal subgroups of H are noncompact.

This theorem is related to the problem of describing the group of all isometries of a connected homogeneous Riemannian manifold M in terms of a given transitive connected subgroup G. Indeed if A is the connected component of the identity in the full isometry group of M, then $A=GL$ where L, the isotropy subgroup of A at a point of M, is compact.

Part (a) of the theorem generalizes and provides a new proof of a result of [1] in which the normality of G in A is established when G itself is semisimple of noncompact type. Following the proof of the theorem, we will note a sufficient condition for equality of the noncompact parts of Levi factors of G and A, generalizing a further result of [1].

2. Recall that all maximal compact subgroups of a connected Lie group A are conjugate under an inner automorphism of A. If $A=GL$ with L compact and if U is a maximal compact subgroup of A, then a conjugate of L lies in U. It is then easily verified that $A=GU$. Thus we are free to replace L by any convenient maximal compact subgroup of A.

A maximal connected semisimple subgroup A_{ss} of A will as usual be called a Levi factor of A. Being semisimple, A_{ss} is a product $A_{ss}=A_{nc}A_c$ of connected normal semisimple subgroups A_{nc} and A_c of noncompact and compact type, respectively. A_{nc} and A_c will be called the noncompact and compact

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