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REMARKS ON ONE-DIMENSIONAL SEMINORMAL RINGS

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Various characterizations of reduced seminormal rings of dimension one are given in Salmon [4], Bombieri [1] and Davis [2]. Among others it is shown that if $(A, \mathfrak{m})$ is a local ring of a closed point on an algebraic curve defined over an algebraically closed field k, then A is seminormal if and only if the completion $\hat{A}$ is k-isomorphic to $k[[X_1, \cdots, X_n]]/(\cdots, X_iX_j, \cdots)$ where $i \neq j$ ([1]) or the associated graded ring $Gr^{\cdot}(A)$ is k-isomorphic to $k[X_1, \cdots, X_n]/(\cdots, X_iX_j, \cdots)$ where $i \neq j$ ([2]). Generalizing these results we prove the following in the first section. Under certain moderate assumptions on A there exist an integer n and an ideal I in $k[X_1, \cdots, X_n]$ such that A is seminormal if and only if $\hat{A} \cong k[[X_1, \cdots, X_n]]/Ik[[X_1, \cdots, X_n]]$ or $Gr^{\cdot}(A) \cong k[X_1, \cdots, X_n]/I$. Moreover the ideal I is generated by quadratic forms and these forms and integer n are determined solely by the k-algebra structure of $\bar{A}/J(\bar{A})$, where $\bar{A}$ is the integral closure of A in the total quotient ring of A and $J(\bar{A})$ is the Jacobson radical of $\bar{A}$. Let C be a plane algebraic curve and let P be a closed point on C. Then it is known that the local ring $O_{P,C}$ is seminormal if and only if P is a simple point or a node (cf. [1], [2], [4]). It is then natural to ask what the seminormalization of $O_{P,C}$ is when P is not a seminormal point. The answer to this question is given in the second section in the case where P is an ordinary multiple point.

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0. Notations and conventions

The following notations and conventions are fixed throughout this article. When R is a ring, $J(R)$ stands for the Jacobson radical of R, $Q(R)$ for the total quotient ring of R, $\bar{R}$ for the integral closure of R in $Q(R)$ and ^+R for the seminormalization of R. We denote by $\cong_R$ an R-algebra isomorphism. An R-algebra is always assumed to be commutative, associative and containing 1. The symbols X, Y, Z, T, X_i, etc. are used to denote indeterminates or variables. When we say that $(R, \mathfrak{M})$ is a quasi-local ring, we mean that R is a ring which has the unique maximal ideal $\mathfrak{M}$. A noetherian quasi-local ring is called a local ring.