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UNISERIAL RINGS AND LIFTING PROPERTIES

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We have studied the lifting (extending) property of simple modules on direct sums of cyclic hollow (uniform) modules in [4] and [5]. We have shown there that they are closely related to generalized uniserial rings, provided that the ring is right and left artinian.

We shall extend those relations to more general rings. In the second section we shall study rings R with the lifting (extending) property of simple modules as right R-modules by making use of results in [6] and [7]. Especially, we shall show that the ring of upper (lower) tri-angular matrices (of column finite) over a division ring with countable degree have the above property. We shall stydy, in the third section, relations between right generalized uniserial (couniserial) rings and the lifting (extending) property of simple modules on direct sum of cyclic hollow (uniform) modules, when R is semi-perfect. In the final section we shall study those problems on a commutative ring. We shall determine the type of modules which have the extending property of simple modules, when R is a Dededind domain and give a characterization for a commutative ring R to have the lifting (extending) property of simple modules for any direct sum of cyclic hollow (uniform) modules. Finally we shall show that if R is noetherian, the property mentioned above is equivalent to the fact: R is a direct sum of artinian serial rings and Dededind domains.

1. Definitions

Throughout this paper we assume that a ring R contains an identity element and every R-module M is a unitary right R-module. We call M a *completely indecomposable* if $\mathrm{End}_R(M)$ is a local ring. We denote the Jacobson radical, injective hull and the socle of M by $J(M)$, $E(M)$ and $S(M)$, respectively. $M^{(J)}$ means the direct sum of J-copies of M and $|M|$, $|J|$ mean the composition length of M and the cardinal of J. We put $S_i(M)/S_{i-1}(M)=S(M/S_{i-1}(M))$ inductively, namely the *lower Loewy series*. By $\bar{M}$ we denote $M/J(M)$. If for any simple submodule A of $\bar{M}$ (resp. $S(M)$) there exists a completely indecomposable cyclic hollow (resp. uniform) direct summand N of M such that $N+J(M)/J(M)=\bar{N}=A$ (resp. A is essential in N (i.e. $A_e\subseteq N$)), we say M has *the lifting* (resp. *extending*) *property of simple modules*. See [4], [5], [6] and [7] for other definitions.