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EQUIVARIANT STABLE HOMOTOPY GROUPS OF SPHERES WITH INVOLUTIONS, I

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Introduction. Equivariant stable homotopy groups of spheres with linear involutions were first discussed by Bredon [4, 5] and then by Landweber [10]. But the precise results of computation are not published except a few number of examples even though it seems that they computed these groups to a certain extent. In the present series of works we try to compute these groups systematically.

We use the notations $\pi_{p,q}^S$ of Landweber [10] to denote these groups which are denoted by $\pi_{p+q,p}^S$ in Bredon [4, 5]. As is well-known there are two types of homomorphisms; the one is the *forgetful homomorphism* ψ: $\pi_{p,q}^S \rightarrow \pi_{p+q}^S$ and the other is the *fixed-point homomorphism* ϕ: $\pi_{p,q}^S \rightarrow \pi_q^S$. There are also exact sequences involving these homomorphisms, i.e.

$$\cdots \rightarrow \pi_{p+q}^S \rightarrow \pi_{p,q}^S \xrightarrow{\chi} \pi_{p-1,q}^S \xrightarrow{\psi} \pi_{p+q-1}^S \rightarrow \cdots$$

and

$$\cdots \rightarrow \pi_{q+1}^S \rightarrow \lambda_{p,q}^S \rightarrow \pi_{p,q}^S \xrightarrow{\phi} \pi_q^S \rightarrow \cdots,$$

which are called *forgetful* and *fixed-point exact sequences*. These were certainly two of basic tools by Bredon and Landweber, and we also use these as a part of our basic tools.

Here we present two isomorphisms; the one is

$$\phi\colon \pi_{p,q}^S \approx \pi_q^S \qquad \text{for } p+q<0,$$

the other is that the fixed-point exact sequence splits in a large part and gives the isomorphism

$$\pi_{p,q}^S \approx \lambda_{p,q}^S \oplus \pi_q^S \qquad \text{for } p<q \text{ or } q<-1.$$

The first isomorphism reduces the computation of $\pi_{p,q}^S$ for $p+q<0$ to that of ordinary stable stems. The second isomorphism reduces the computation of

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