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BLOCK INTERSECTION NUMBERS OF BLOCK DESIGNS

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1. Introduction

Let t, v, k and λ be positive integers with $v\geqslant k\geqslant t$. A $t-(v, k, \lambda)$ design is a pair consisting of a v-set Ω and a family $\boldsymbol{B}$ of k-subsets of Ω, such that each t-subset of Ω is contained in λ elements of $\boldsymbol{B}$. Elements of Ω and $\boldsymbol{B}$ are called points and blocks, respectively. A $t-(v, k, \lambda)$ design is called nontrivial provided $\boldsymbol{B}$ is a proper subfamily of the family of all k-subsets of Ω, then $t<k<v$. In this paper, we assume that all designs are nontrivial. For a $t-(v, k, \lambda)$ design $\boldsymbol{D}$ we use λ_i $(0\leqslant i\leqslant t)$ to represent the number of blocks which contain a given set of i points of $\boldsymbol{D}$. Then we have

$$\lambda_i=\frac{\binom{v-i}{t-i}}{\binom{k-i}{t-i}}\lambda=\frac{(v-i)(v-i-1)\cdots(v-t+1)}{(k-i)(k-i-1)\cdots(k-t+1)}\lambda \qquad (0\leqslant i\leqslant t)\,.$$

A $t-(v, k, \lambda)$ design $\boldsymbol{D}$ is called block-schematic if the blocks of $\boldsymbol{D}$ form an association scheme with the relations determined by size of intersection (cf. [3]). In §2, we prove the following theorem which extends the result in [1].

Theorem 1. (a) *For each $n\geqslant 1$ and $\lambda\geqslant 1$, there exist at most finitely many block-schematic $t-(v, k, \lambda)$ designs with $k-t=n$ and $t\geqslant 3$.*

(b) *For each $n\geqslant 1$ and $\lambda\geqslant 2$, there exist at most finitely many block-schematic $t-(v, k, \lambda)$ designs with $k-t=n$ and $t\geqslant 2$.*

REMARK. Since there exist infinitely many $2-(v, 3, 1)$ designs and since every $2-(v, k, 1)$ design is block-schematic (cf. [2]), Theorem 1 does not hold for $\lambda=1$ and $t=2$.

For a block B of a $t-(v, k, \lambda)$ design $\boldsymbol{D}$ we use $x_i(B)$ $(0\leqslant i\leqslant k)$ to denote the number of blocks each of which has exactly i points in common with B. If, for each i $(i=0, \cdots, k)$, $x_i(B)$ is the same for every block B, we say that $\boldsymbol{D}$ is block-regular and we write x_i instead of $x_i(B)$. We remark that if a $t-(v, k, \lambda)$ design $\boldsymbol{D}$ is block-schematic then $\boldsymbol{D}$ is block-regular. For any $t-(v, k, 1)$ design or any $t-(v, t+1, \lambda)$ design, either of which is block-regular (cf. Lemma 1),