Harada, M. and Oshiro, K.
Osaka J. Math.
18 (1981), 767–785

ON EXTENDING PROPERTY ON DIRECT SUMS OF UNIFORM MODULES

Dedicated to Professor Kentaro Murata on his 60th birthday

MANABU HARADA AND KIYOICHI OSHIRO

(Received February 18, 1980)

First we take a right artinian ring R. Then every injective R-module E is a direct sum of indecomposable modules. Further for every simple submodule S of E, there exists a direct summand of E whose socle is equal to S. Let $\sum_I \oplus S_\alpha$ be a decomposition of the socle of E. Then we have a decomposition of E by indecomposable modules E_α such that $E=\sum_I \oplus E_\alpha$ and the socle of E_α is S_α. We shall call the first property and the second propert the extending property of simple module and of decomposition, respectively. These concepts are dual to those of lifting properties mentioned in [7].

We shall study the above properties on direct sums of completely indecomposable modules with certain condition over an arbitiary ring. We shall give characterizations of those properties in terms of endomorphisms over direct summands and show that quasi-injective modules and generalized uniserial rings [15] are related to those properties. Our results are dual or similar to those in [9] and are applied to the study of QF-2 rings in [8].

1. Notations

Throughout this paper R is a ring with identity and every R-module is a unitary right R-module. For an R-module M, we denote its socle and its injective envelope by $S(M)$ and $E(M)$, respectively. For a submodule N of M, we use the symbol $N\subseteq_e M$ to indicate that M is an essential extension of N.

In [9], the first author has studied direct sums of hollow modules by introducing the lifting property of simple module and that of decomposition. In order to deal with their dual properties, we must consider the dual condition to (E-I) in [9]:

(M-I) *Every monomorphism of an R-module into itself is isomorphic.*

If a uniform R-module M satisfies (M-I), then $\mathrm{End}_R(M)$ is a local ring, namely M is *completely indecomposable*. In particular, indecomposable quasi-injective R-modules are completely indecomposable modules with (M-I). Artinian R-modules clearly satisfy (M-I).