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ON p-RADICAL DESCENT OF HIGHER EXPONENT

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0. Introduction

In the paper [8], P. Samuel has developed the theory of p-radical descent of exponent one by making use of logarithmic derivatives. In this article we shall give a generalization of his theory to the case of p-radical descent of higher exponent with the aid of a finite set of higher derivations of finite rank.

In the first section some preparatory results are collected. Let A be a Krull domain of characteristic $p>0$ and K be its quotient field. Let $\boldsymbol{D}=(\underline{D}^{(1)}, \cdots, \underline{D}^{(r)})$ be an r-tuple of non-trivial higher derivations $\underline{D}^{(i)}$'s of rank m_i on K which leave A invariant. For simplicity we shall abuse the notation $\underline{D}^{(i)}$ to denote the ring homomorphism of K into a truncated polynomial ring of order m_i over K, i.e., $K[t_i: m_i]: =K[T_i]/T_i^{m_i+1}$ associated to the higher derivation $\underline{D}^{(i)}$. Let K' be the intersection of the fields of $\underline{D}^{(i)}$-constants $(1\leq i\leq r)$ and let $A': = A\cap K'$. Let $\boldsymbol{T}=(T_1, \cdots, T_r)$ be an r-ruple of indeterminates and let t_i be the residue class of T_i modulo $T_i^{m_i+1}$ in $K[T_i]/T_i^{m_i+1}$. We shall set $\boldsymbol{t}: =(t_1, \cdots, t_r)$ and $\boldsymbol{m}: =(m_1, \cdots, m_r)$. We shall denote $\prod_{i=1}^{r} K[t_i: m_i]$ by $K[\boldsymbol{t}: \boldsymbol{m}]$. Similarly we denote $\prod_{i=1}^{r} A[t_i: m_i]$ by $A[\boldsymbol{t}: \boldsymbol{m}]$ where $A[t_i: m_i]$ is a truncated polynomial ring of order m_i over A. Furthermore we shall define a ring homomorphism $\boldsymbol{D}$ of K into $K[\boldsymbol{t}: \boldsymbol{m}]$ by $\boldsymbol{D}(z)=(\underline{D}^{(1)}(z), \cdots, \underline{D}^{(r)}(z))$ $(z\in K)$. Let $\mathcal{L}_A$ and $\mathcal{L}'_A$ be the sets of elements defined respectively by

$$\mathcal{L}_A = \{\boldsymbol{D}(z)/z\in K[\boldsymbol{t}: \boldsymbol{m}] \mid z\in K^*,\ \boldsymbol{D}(z)/z\in A[\boldsymbol{t}: \boldsymbol{m}]\}\ ,$$
$$\mathcal{L}'_A = \{\boldsymbol{D}(u)/u \mid u\in A^*\}\ .$$

Let $\boldsymbol{j}$: $\mathrm{Div}(A') \rightarrow \mathrm{Div}(A)$ be the homomorphism defined by $\boldsymbol{j}(\mathcal{Q})=e(\mathcal{P})\mathcal{P}$ where, $\mathcal{Q}$ is a prime ideal of height one in A', $\mathcal{P}$ is the unique prime ideal of height one in A with $\mathcal{P}\cap A'=\mathcal{Q}$ and $e(\mathcal{P})$ is the ramification index of $\mathcal{P}$ over $\mathcal{Q}$. Then we can define the homomorphism $\bar{\boldsymbol{j}}$: $\mathrm{Cl}(A')\rightarrow \mathrm{Cl}(A)$ induced by $\boldsymbol{j}$ (cf. [8]). Let $\mathcal{D}$ be the subgroup of $\mathrm{Div}(A')$ consisting of divisors E's such that $\boldsymbol{j}(E)$ is principal and let Φ_0: $\mathcal{D}\rightarrow \mathcal{L}_A/\mathcal{L}'_A$ be the homomorphism defined by $\Phi_0(E)=\boldsymbol{D}(x)/x$ modulo $\mathcal{L}'_A$, where $E\in\mathcal{D}$ and $\boldsymbol{j}(E)=\mathrm{div}_A(x)$. Let Φ: $\mathrm{Ker}(\bar{\boldsymbol{j}})=\mathcal{D}/F(A')\rightarrow \mathcal{L}_A/\mathcal{L}'_A$ be the homomorphism induced by Φ_0 where $F(A')$ denotes the subgroup of $\mathrm{Div}(A')$