Ikegami, T.
Osaka J. Math.
18 (1981), 699–702

ON A GENERALIZATION OF LUKEŠ' THEOREM

TERUO IKEGAMI

(Received March 25, 1980)

0. In the classical potential theory, M.V. Keldych [4] proved the existence of positive harmonic function h in a bounded domain D possessing the property that h can be extended continuously onto $\bar{D}$ and takes the value zero only at the prescribed regular boundary point. In virtue of this result it is possible to obtain interesting results such as characterization of generalized Dirichlet solutions H_f by its functional aspect and identity of the set of regular boundary points with the Choquet boundary. In the axiomatic framework, the former problem was studied by J. Lukeš [5] thoroughly and it was cleared up that for a relatively compact open set the classical result holds as well only when the axiom of polarity is assured. As for the latter one, Bliedtner-Hansen [1] proved completely in their deep result that it remains valid under some condition even in an open set not necessarily relatively compact. In view of this, it seems to be adequate to investigate the former problem for an arbitrary open set. The purpose of this note is to extend Lukeš' theorem to an open set and normalized solutions of Dirichlet problem.

1. Let X be a $\mathcal{P}$-harmonic space with countable base in the sense of [3], and let P be the set of all continuous potentials on X. We consider an open subset U of X with non-empty boundary ∂U. We define

$C_P(E)\colon =\{f;\ f \text{ is continuous on } E,\ \exists p\in P \text{ such that } |f|\leq p\}$ for $E\subset X$,

$S(U)\colon =\{s\in C_P(\bar{U});\ s \text{ is superharmonic on } U\}$.

For an extended real valued function f on ∂U, we consider

$$\bar{H}_f^0(a)\colon =\inf\left\{v(a);\ \begin{array}{l}\text{hyperharmonic on } U, \text{ bounded below,}\\ \liminf v\geq f \text{ on } \partial U,\ v\geq 0 \text{ outside a compact}\\ \text{subset of } X\end{array}\right\}$$

and

$\underline{H}_f^0\colon =-\bar{H}_{(-f)}^0$

If $\bar{H}_f^0=\underline{H}_f^0$ and is harmonic, f is called resolutive and $\bar{H}_f^0=\underline{H}_f^0=H_f^0$ is called a normalized solution. It is known that all functions of $C_P(\partial U)$ are resolutive, thus, for each $a\in U$ there exists a Borel measure λ_a such that

$$\lambda_a(f) = H_f^0(a) \qquad \text{for every } f\in C_P(\partial U)\,.$$