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EQUIVARIANT DESUSPENSION OF G-MAPS

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1. Introduction

In this paper we will give sufficient conditions for a G-map to desuspend equivariantly. Throughout this paper G always denotes a compact Lie group.

For a G-space M let M^{Σ} be the unreduced suspension defined to be the quotient space of $M\times[0,1]$ in which $M\times\{0\}$ is collapsed to one point (called the south pole) and $M\times\{1\}$ is collapsed to another point (called the north pole). Giving the trivial G-action on $[0,1]$, a G-action on M^{Σ} is naturally induced. The unreduced suspension $f^{\Sigma}: M^{\Sigma}\to N^{\Sigma}$ of a G-map $f: M\to N$ is also a G-map.

If H is a closed subgroup of G, then (H) and $N(H)$ denote the conjugacy class and the normalizer of H in G, respectively. For a point x of a G-space M, G_x denotes the isotropy subgroup of G at x. The conjugacy class of an isotropy subgroup is called an isotropy type on M. Define $\mathcal{I}(M)$ to be the set of all isotropy types on M. Define

$$M^H = \{x\in M \mid H\subset G_x\}\ .$$

If M is a smooth G-manifold, then M^H is an $N(H)$-invariant submanifold of M, which possibly has various dimensional components. Define dim M^H to be the maximum of those dimensions.

The main result of this paper is:

Theorem. *Let M be a compact, smooth G-manifold, and N a G-space. Let $f: M^{\Sigma}\to N^{\Sigma}$ be a G-map such that $f(z_\varepsilon)=z'_\varepsilon$ for $\varepsilon=0,1$, where z_0 and z_1 are the south pole and the north pole of M^{Σ} respectively, and z'_0 and z'_1 are those of N^{Σ}. Suppose that for all $(H)\in\mathcal{I}(M)$ there are non-negative integers n_H satisfying the following conditions:*

(i) $\dim M^H - \dim N(H)/H \le n_H+1$,

(ii) *N^H is n_H-connected, and*

(iii) *if $n_H=0$, $\pi_1(N^H)$ is abelian.*

Then f is G-homotopic to h^{Σ} relative to $\{z_0, z_1\}$ for some G-map $h: M\to N$.

$S(V)$ denotes the unit sphere in an orthogonal representation V of G. $\boldsymbol{R}$ denotes the trivial one-dimensional representation of G. Then $S(V\oplus\boldsymbol{R})$ may