

## ISOTROPIC SUBMANIFOLDS WITH PARALLEL SECOND FUNDAMENTAL FORM IN $P^n(c)$

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**Introduction.** In the study of submanifolds of a riemannian manifold, as a generalization of a totally geodesic submanifold, the notion of an isotropic submanifold has been introduced by B. O'Neill [10]. On the other hand, as another generalization of a totally geodesic submanifold, there is the notion of a submanifold with parallel second fundamental form. Among submanifolds belonging to both classes, those which are not totally geodesic have the property that every geodesic in the submanifold is a circle in the ambient riemannian manifold (K. Nomizu [8]).

These submanifolds have been studied recently when the ambient riemannian manifold is a riemannian symmetric space. Among them totally umbilical submanifolds are called extrinsic spheres. It is known that an extrinsic sphere is isometric to a Euclidean sphere, a Euclidean space, or a real hyperbolic space (B.Y. Chen [2] and H. Naitoh [7]). If the ambient manifold is a Hermitian symmetric space, a Kähler submanifold belonging to both classes is congruent to the Veronese manifold of degree two (H. Naitoh [7]). Moreover K. Nomizu [8] has shown that if the ambient manifold is a complex projective space with the Fubini-Study metric, the Veronese manifold of degree two is characterized by the property that every geodesic in the submanifold is a circle in the complex projective space.

Now nonzero isotropic submanifolds with parallel second fundamental form are closely related to planar geodesic submanifolds. When the ambient manifold is a Euclidean sphere, the submanifolds coincide with those which are planar geodesic but not totally geodesic, and they have been classified by K. Sakamoto [12]. When the ambient manifold is the complex projective space, submanifolds which are planar geodesic but not totally geodesic are nonzero isotropic and have parallel second fundamental forms. Moreover it is known that these submanifolds are compact riemannian symmetric spaces of rank one (J.S.Pak [11]).

In this paper we study nonzero isotropic submanifolds with parallel second fundamental form in a complex projective space with the Fubini-Study metric. These submanifolds can be divided into the following three types; Kählerian,