Tsukamoto, C.
Osaka J. Math.
18 (1981), 407-426

SPECTRA OF LAPLACE-BELTRAMI OPERATORS ON $SO(n+2)/SO(2)\times SO(n)$ AND $Sp(n+1)/Sp(1)\times Sp(n)$

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(Received October 15, 1979)

Introduction. Let $M=G/K$ be a compact symmetric space with G compact and semisimple. We assume that the Riemannian metric on M is the metric induced from the Killing form sign-changed. We consider the Laplace-Beltrami operator Δ^p acting on p-forms and its spectrum $\mathrm{Spec}^p(M)$.

Ikeda and Taniguchi [3] computed $\mathrm{Spec}^p(M)$ for $M=S^n$ and $P^n(C)$, studying representations of G and K. They showed that $\Delta^p=-$Casimir operator when we consider the space of p-forms $C^\infty(\Lambda^p M)$ as a G-module. Each irreducible G-submodule of $C^\infty(\Lambda^p M)$ is included in some eigenspace of Δ^p and the sum of irreducible G-submodules of $C^\infty(\Lambda^p M)$ equals to the sum of eigenspaces of Δ^p. We can compute eigenvalues from Freudenthal's formula and multiplicities from Weyl's dimension formula. Thus to compute $\mathrm{Spec}^p(M)$, we have only to decompose $C^\infty(\Lambda^p M)$ into irreducible G-submodules and count out them.

But generally it is not easy. Though Beers and Millman [1] determined $\mathrm{Spec}^p(M)$ when M is a Lie group of a low rank such as $SU(3)$ or $SO(5)$ by the similar method, these seem to be all we know.

Frobenius' reciprocity law enables us to reduce the problem into the following two: How does an irreducible G-module decompose into irreducible K-modules? How does the p-th exterior product of (complexified) cotangent space decompose into irreducible K-modules? The former is usually called a branching law.

In this paper, we give a branching law for $G=SO(n+2)$ and $K=SO(2)\times SO(n)$, which enables us to compute $\mathrm{Spec}^p(M)$. As a matter of fact, we should distinguish between the case $n=$odd and the case $n=$even. Almost in parallel, we get a branching law for $G=Sp(n+1)$ and $K=Sp(1)\times Sp(n)$, which reproduces the result of Lepowsky [4] obtained in a different way.

The latter problem, i.e., the decomposition of an exterior power of an isotropy representation is a rather technical (but indispensable) part in computing $\mathrm{Spec}^p(M)$. We give a complete list of members in the decomposition for $G=SO(n+2)$ and $K=SO(2)\times SO(n)$. For $G=Sp(n+1)$ and $K=Sp(1)\times Sp(n)$, we confine ourselves to indicating a procedure to determine the decom-