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ON THE DEGREES OF IRREDUCIBLE REPRESENTATIONS OF FINITE GROUPS

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1. Introduction

Let G be a finite group of order $|G|$ and F be an algebraically closed field of characteristic 0. Let T be an irreducible representation of G over F and d_T be the degree of T. As is well know, d_T divides $|G|$. Furthermore there exists a sharper result due to Ito [2], namely, d_T divides the index in G of every abelian normal subgroup of G. Let s_T be the order of det T, that is, s_T is the smallest natural number such that $|T(x)|^{s_T}=1$ for all $x \in G$, where $|T(x)|$ is the determinant of $T(x)$. In Lemma of [4] we showed the first part of the following

Theorem 1. *Let T be an irreducible representation of G over F. Then we have*

(i) $d_T s_T \bigm| 2|G|$,
(ii) *if d_T or s_T is odd then* $d_T s_T \bigm| |G|$.

The second part follows from (i) by considering the 2-part of $d_T s_T$, since both d_T and s_T divide $|G|$.

The purpose of the present paper is to prove the following theorems.

Theorem 2. *If G has an irreducible representation T over F with $d_T s_T \nmid |G|$, then the following holds.*

(i) *A 2-Sylow subgroup P of G is cyclic and $P \neq 1$. Hence G has the normal 2-complement K.*
(ii) $C_P(K)=1$.
(iii) *T is induced from a representation of K.*

The converse of Theorem 2 is also true:

Theorem 3. *If G satisfies* (i) *and* (ii) *in Theorem* 2, *then G has an irreducible representation T with $d_T s_T \nmid |G|$.*

We also have the following

Theorem 4. *Let T be an irreducible representation of G over F. Then we have*