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A FAMILY OF FOURIER INTEGRAL OPERATORS AND THE FUNDAMENTAL SOLUTION FOR A SCHRÖDINGER EQUATION*

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Introduction

In this paper we shall study the theory of Fourier integral operators on R^n depending on a parameter $h\in(0,1)$ with non-homogeneous phase functions and certain symbols in sections 1–4, and apply this theory to the construction of the fundamental solutions for the Cauchy problem of a pseudo-differential equation of Schrödinger's type in sections 5 and 6.

In section 1 we shall study a calculus of a family of pseudo-differential operators $P_h=p_h(X,D_x)$ with C^∞-symbols $p_h(x,\xi)$ depending on a parameter $h\in(0,1)$, which is defined by

$$P_h u(x)=\int e^{ix\cdot\xi}p_h(x,\xi)\hat{u}(\xi)đ\xi,\quad u\in\mathscr{S}, \tag{1}$$

where $đ\xi=(2\pi)^{-n}d\xi$, $\hat{u}(\xi)$ denotes the Fourier transform of u, and $\mathscr{S}$ denotes the Schwartz space of rapidly decreasing functions on R^n. Let $\mathscr{B}(R^{2n})$ be the space of C^∞-functions in R^{2n} whose derivatives of any order are all bounded in R^{2n}. Then, the symbols $p_h(x,\xi)$ are defined as those functions which satisfy

$$\text{“}\{h^{-m-\rho|\alpha|+\delta|\beta|}D_x^\beta\partial_\xi^\alpha p_h(x,\xi)\}_{0<h<1}\text{ is bounded in }\mathscr{B}(R^{2n})\text{”} \tag{2}$$

for any α, β with some $-\infty<m<\infty$ and $0\leqq\delta\leqq\rho\leqq 1$, and we denote this symbol class by $B^m_{\rho,\delta}(h)$.

In section 2 we shall first define a class $P(\tau,l)$ of phase functions with $0\leqq\tau<1$ and an integer $l\geqq 0$ as the class of C^∞-functions such that $J(x,\xi)\equiv\phi(x,\xi)-x\cdot\xi$ satisfies

$$\begin{aligned}|J|_l\equiv&\sum_{|\alpha+\beta|\leqq 1}\sup_{x,\xi}\{|D_x^\beta\partial_\xi^\alpha J(x,\xi)|/\langle x;\xi\rangle^{2-|\alpha+\beta|}\}\\&+\sum_{2\leqq|\alpha+\beta|\leqq 2+l}\sup_{x,\xi}\{|D_x^\beta\partial_\xi^\alpha J(x,\xi)|\}\leqq\tau\\&\qquad(\langle x;\xi\rangle=(1+|x|^2+|\xi|^2)^{1/2})\end{aligned} \tag{3}$$

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