Nishioka, K.
Osaka J. Math.
18 (1981), 249-255

A THEOREM OF PAINLEVÉ ON PARAMETRIC SINGULARITIES OF ALGEBRAIC DIFFERENTIAL EQUATIONS OF THE FIRST ORDER

KEIJI NISHIOKA

(Received September 7, 1979)

0. Introduction

Consider an algebraic differential equation of the first order $F(y,y')=0$ over an algebraically closed ordinary differential field k of characteristic 0, where F is an irreducible polynomial over k. Recently Matsuda [3] presented a differential-algebraic definition for $F=0$ to be free from parametric singularities and gave a purely algebraic proof of the following theorem essentially due to Fuchs [2] and Poincaré [8]: Suppose that $F=0$ is free from parametric singularities. Then it is reduced to a Riccati equation or a defining equation of elliptic function by a birational transformation over k if the genus of $F=0$ is 0 or 1 respectively. The author [4] proved that under the above assumption it is reduced to an equation of Clairaut type by a birational transformation over k if the genus is greater than 1. This theorem is essentially due to Poincaré [8], Painlevé [5] and Picard [6].

Here a differential-algebraic formulation and its proof of the following theorem which is essentially due to Painlevé [5], [6] will be given: The general solution η of $F=0$ depends algebraically upon an arbitrary constant over some differential extension field of k if and only if there exists an algebraic differential equation of the first order $G=0$ over k such that it is free from parametric singularities and the general solution of $G=0$ is a rational function of η and η' over k. Here, we assume that k contains non-constants.

Let k be an algebraically closed ordinary differential field of characteristic 0, and Ω be a universal differential extension field of k. Suppose that K is a differential subfield of Ω and it is an algebraic function field of one variable over k. Let P be a prime divisor of K and K_P be the completion of K with respect to P. Then K_P is a differential extension field of K and the differentiation is continuous in the metric of K_P (cf. [1, p. 114]). Let ν_P and t_P denote respectively the normalized valuation belonging to P and a prime element in P. The following definition is due to Matsuda [3]: K is said to be *free from parametric singularities* over k if we have $\nu_P(t'_P)\geqq 0$ for each prime divisor P of