Kado, J.
Osaka J. Math.
18 (1981), 55-61

UNIT-REGULAR RINGS AND SIMPLE SELF-INJECTIVE RINGS

JIRO KADO*)

(Received October 8, 1979)

Unit-regular algebras over a field are investigated from the point of view of a directed abelian group with order-unit.

In the section 1 we show that if $(K_0(R), [R])$ is an ultrasimplicial abelian group for a unit-regular algebra R over a field F, then R has a subalgebra T such that T is an ultramatricial F-algebra and R is generated as a ring by T and units of R.

In the section 2 we discuss a simple left and right self-injective ring R which is not artinian. Let F be the center of R and F_∞ be the completion of a ring which is a direct limit of $M_2(F)\rightarrow M_{2^2}(F)\rightarrow\cdots$, where homomorphisms are diagonal maps. We show that there exists a subalgebra S of R such that S is isomorphic to F_∞ as a F-algebra and that every idempotent of R is conjugate to an idempotent of S and that every element of R has the form uev, where u, v are units in R and e is an idempotent of S.

We take most of our terminologies and notations from Goodearl's recent book [3], and rely as well on this work for statements of known results.

Throughout this paper a ring is an associative ring with identity and modules are unitary

1. Unit-regular algebras

DEFINITION [2]. A ring R is *unit-regular* if for each $x\in R$ there is some unit (i.e. invertible element) $u\in R$ such that $xux=x$.

DEFINITION [3, p. 200]. For any ring R the *Grothendieck group* $K_0(R)$ is an abelian group with generators $[A]$, where A is any finitely generated projective right R-modules, and with relations $[A]+[B]=[C]$ whenever $A\oplus B\cong C$. Two generators $[A]$, $[B]$ equal in $K_0(R)$ if and only if $A\oplus R^n\cong B\oplus R^n$ for some positive integer n. Every element of $K_0(R)$ has the form $[A]$-$[B]$ for suitable modules A, B.

*) Supported by Grant-in-Aid for Scientific Research, No. 454017