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ON LIFTING PROPERTY ON DIRECT SUMS OF HOLLOW MODULES

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Following E. Mares [12] and H. Bass [2] we shall first consider a semi-perfect module P over a ring R. One of the important properties of P is the lifting property as follows: Let $P/J(P)=\sum_I \oplus K_\alpha$ be a decomposition of $P/J(P)$, then there exists a decomposition of P: $P=\sum_I \oplus P_\alpha$ such that $\varphi(P_\alpha)=K_\alpha$ for all $\alpha \in I$, where $J(P)$ is the Jacobson radical of P and φ is the natural epimorphism of P onto $P/J(P)$. In case the module is injective, we have studied irredundant sum of indecomposable injective modules and the lifting property of decomposition over a perfect ring satisfying a certain condition in [7].

In this note we shall generalize those properties over an arbitrary ring. In order to do so, it is quite natural to take a module M_α such that $M_\alpha/J(M_\alpha)$ is a simple module instead of P_α, namely a hollow module [3]. For a direct sum of hollow modules M we shall give some characterizations of the lifting property of simple module and of decomposition of M (see the definition in §1). Finally, we shall give characterizations of artinian rings with lifting property (namely, generalized uniserial ring and semi-simple ring). We shall study the dual property -the extending property- of simple module in [8].

1. Definitions

Throughout this paper we consider a ring R with identity and we assume every module M is a unitary right R-module. We shall denote the Jacobson radical of M by $J(M)$.

Let $\{M_\alpha\}_I$ be a set of submodules of M. If $M=\sum_I M_\alpha$ and $M \neq \sum_J M_\beta$ for any proper subset J of I, we call $\sum_I M_\alpha$ *be an irredundant sum* [7]. If $\sum_K M_\gamma$ is a direct summand of M for every finite subset K of I, we say $\sum_I M_\alpha$ be a *locally direct summand* of M [9]. We denote the natural epimorphism of M onto $M/J(M)$ by φ. If there exists a direct summand M_α of M such that $\varphi(M_\alpha)=A_\alpha$ for each simple submodule A_α of $M/J(M)$, then we say M have the *lifting property of simple module.*