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# ON THE SPECTRUM OF A RIEMANNIAN MANIFOLD OF POSITIVE CONSTANT CURVATURE II

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**Intoroduction.** A complete connected riemannian manifold of positive constant curvature 1 is called a Clifford-Klein spherical space form. In this paper, we call such a riemannian manifold simply a *spherical space form*. An $n$-dimensional spherical space form is obtained as $S^n/G$, the standard unit sphere $S^n$ modulo a finite group of fixed point free isometries $G$. In the previous paper [4], we raised the problem,

"*Does the spectrum of a spherical space form determine the spherical space form among all spherical space forms?*"

The above problem was solved affirmatively first for a 3-dimensional lens space $M$ by Tanaka [6] in case where the order of fundamental group of $M$, $|\pi_1(M)|$ is prime or 2-times prime, by the author and Yamamoto in case where $|\pi_1(M)|$ is prime power or 2-times prime power, and by Yamamoto [11] in case where $|\pi_1(M)|$ is any composite number. For any other 3-dimensional spherical space forms and homogeneous space forms, it was also affirmatively solved recently by the author [5].

In this paper, we shall attack the above problem for the spherical space form with dimension of the form $4k+1$ ($k\geqq 1$). Main tool in this paper is also the generating function associated to the spectrum of a spherical space form $S^{4k+1}/G$ constructed in [4] and [5]. The relations between the finite group $G$ and the generating function were studied in [5]. Here, we investigate the relations more details for $(4k+1)$-dimensional spherical space forms using the complete classification of the manifolds due to Vincent [9] (see also [10]).

Our main results are the followings.

**Theorem 3.1.** *Let $d$ be an odd prime. Let $M$, $N$ be $(2d-1)$-dimensional spherical space forms. Suppose $M$ is isospectral to $N$. Then their fundamental groups are isomorphic.*

**Theorem 3.9.** *Let $M$ be a 5-dimensional spherical space form with non-*