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ON THE GROUPS $J_{Z_{m,q}}(*)$

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1. Introduction

Let G be a compact topological group. If V is an orthogonal representation space of G, we denote by $S(V)$ its unit sphere with respect to some G-invariant inner product. Two orthogonal representation spaces V and W of G are called *J-equivalent* if there exists an orthogonal representation space U such that $S(V\oplus U)$ and $S(W\oplus U)$ are G-homotopy equivalent. Let $RO(G)$ denote the real representation ring of G, and let $T_G(*)\subset RO(G)$ denote an additive subgroup consisting of all elements $V-W$ such that V and W are J-equivalent.

In [6] and [7], Kawakubo considered the quotient group $J_G(*)=RO(G)/T_G(*)$ and the natural epimorphism $J_G\colon RO(G)\to J_G(*)$, and determined the structure of $J_G(*)$ for compact abelian topological groups G.

The purpose of this paper is to determine $J_G(*)$ in case G is the metacyclic group

$$Z_{m,q}=\{a, b\,|\,a^m=b^q=e,\ bab^{-1}=a^r\}\ ,$$

where m is a positive odd integer, q is an odd prime integer, $(r-1, m)=1$ and r is a primitive q-th root of 1 mod m. Our main results are Theorem 7.3 and Corollary 7.4.

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2. The metacyclic group $Z_{m,q}$

In this section we recall some well-known results about the metacyclic group $Z_{m,q}$. The metacyclic group $Z_{m,q}$ is a non-abelian group of order mq and every element of $Z_{m,q}$ is written in the form

$$g=a^ib^j\ ,\qquad 0\leqq i\leqq m-1,\quad 0\leqq j\leqq q-1\ .$$

Let $m=p_1^{r(1)}p_2^{r(2)}\cdots p_t^{r(t)}$ be a prime decomposition of m. We can check easily from the definition of $Z_{m,q}$ the following: