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ON THE HOMOTOPY GROUP $\pi_{2n+9}(U(n))$ FOR $n \geqq 6$

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The homotopy groups $\pi_{2n+i}(U(n))$ of the unitary group $U(n)$ for $0 \leqq i \leqq 8$, $i=10$ and 12 were determined by Borel and Hirzebruch [2], Bott [3], Kervaire [7], Toda [22, 23], Matsunaga [8–12], Mimura and Toda [13], Mosher [14, 15], and Imanishi [6]. For $n \geqq 5$ and $i=9$, 11 or 13 the odd components were determined by [12] and [6], but the 2-component had not been completely determined. Indeed Mosher [15] has not determined some group extensions which appear in case of $i=9$ only if $n \equiv 2$, 4 or 6 mod (8) and $n \geqq 6$. In this note we shall determine these group extensions for $i=9$. $\pi_{2n+9}(U(n))$ for $n \leqq 5$ was determined by [6], [13], [15] and [23]. Therefore we shall complete the computation of $\pi_{2n+9}(U(n))$. While the group $\pi_{2n+9}(U(n))$ has been computed by Vastersavendts [24] for $n \equiv 0$ mod (4), 6 mod (8) or 2 mod (16), her results contradict Mosher's [15] and ours for $n \equiv 0$ mod (16) and $n \equiv 6$ mod (8) respectively.

We shall prove

Theorem. *The 2-component cf $\pi_{2n+9}(U(n))$ foi $n \equiv 2, 4$ or 6 mod (8) and $n \geqq 6$ is given by the following table:*

n mod ( )	$\pi_{2n+9}(U(n))$
2(16)	$Z_2 \oplus Z_4 \oplus Z_2$
10(32)	$Z_2 \oplus Z_4 \oplus Z_4$
26(64)	$Z_2 \oplus Z_4 \oplus Z_8$
58(64)	$Z_2 \oplus Z_4 \oplus Z_{16}$
4(8)	$Z_2 \oplus Z_2 \oplus Z_8$
6(8)	$Z_2 \oplus Z_4$

where $Z_m = Z/mZ$ is the cyclic group of order m.

We shall use the notations and terminologies defined in [20] or the book of Toda [23] without any reference.

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