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ON THE IRREDUCIBILITY OF 2-FOLD BRANCHED COVERS OF S^3

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0. Introduction. Montesinos [8] and Hilden [2] showed that every closed, orientable 3-manifold is a 3-fold irregular covering space of S^3 branched over a link ℓ. And Waldhausen [10] showed that two homotopy equivalent closed orientable, sufficiently large 3-manifolds are homeomorphic. So we study that what kind of 3-manifold is irreducible i.e. an embedded 2-sphere in the 3-manifold bounds a 3-ball. Using the result of Montesinos [8] and the surgery technique, we obtain the following.

Theorem. *Let $\ell=k_1\cup\cdots\cup k_\mu$ be a link in S^3 such that every component k_i $(i=1, 2, \cdots, \mu)$ of ℓ is a trivial knot. If $M(\ell)$ is a 2-fold covering space of S^3 branched over ℓ and if $\pi_2(M(\ell))=0$, $M(\ell)$ is irreducible.*

And in section 2 we study a method of determining whether $\pi_2(M(l))=0$ or not for a given link ℓ whose components are all trivial knots.

1. Proof of Theorem

Lemma 1. *Let k be a trivial knot in S^3. If B^3 is a 3-ball in S^3 such that the intersection $\mathscr{D}^1$ of B^3 with k is homeomorphic to the 1-ball, the pair $(B^3, \mathscr{D}^1)$ is a standard pair (i.e. there is an orientation preserving homeomorphism $h:(B^3, \mathscr{D}_1)\to(D^1\times D^2, D^1\times\{0\})$ where D^n is the standard n-ball.*

Proof. Since k is a trivial knot, there is an embedded 2-ball B^2 in S^3 with $\partial B^2=k$. We may assume that B^2 meets ∂B^3 transversally and so $B^2\cap\partial B^3=\{a\ simple\ arc\}\cup\{simple\ closed\ curves\}$. Let α be a simple closed curve in $B^2\cap\partial B^3$ which is innermost in $B^?$ with respect to $B^2\cap\partial B^3$. α splits ∂B^3 into two 2-balls. Let B_α be one of the two 2-balls such that B_α does not contain the simple arc in $B^2\cap\partial B^3$. Since α is innermost in B^2 with respect to $B^2\cap\partial B^3$, there is a 2-ball B'_α in B^2 with $B'_\alpha\cap B^3=\partial B'_\alpha=\alpha$. Then $B_\alpha\cup_\partial B'_\alpha=S^2$ and so $B_\alpha\cup_\partial B'_\alpha$ bounds a 3-ball. Hence there is an ambient isotopy $\{\phi_t\}: S^3\to S^3$ $(0\leqq t\leqq 1)$ keeping α fixed such that $\phi_0=id.$, $\phi_1(B'_\alpha)=B_\alpha$. We may assume that the support of $\{\phi_t\}$ is a small neighborhood of "one of" 3-balls bounded