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ON ONE-SIDED QF-2 RINGS II

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We have studied the extending property on direct sums of indecomposable modules in [4]. We shall apply those results to projective modules and give characterizations of semi-perfect rings whose projective modules have the extending property of simple module. We shall deal with the dual concept of [5].

1. Preliminaries

Throughout this paper we shall denote a ring with identity by R and every R-module M is a right unitary R-module. By $S(M)$ we denote the *socle* of M. We shall recall the definition of extending property of simple module. If for every simple submodule A_α of $S(M)$ there exists a direct summand M_α of M such that $S(M_\alpha)=A_\alpha$, we say M have the *extending property of simple module*. Let $\{N_\beta\}_I$ be a set of submodules of M. If $\bigcap_{I_1} N_\gamma \supsetneqq \bigcap_{I_2} N_\delta$ for subset $I_1 \subsetneqq I_2$, $\bigcap_I N_\delta$ is called *irredundant*.

In this paper we shall study the dual properties to those in [5] and so we shall first introduce the dual condition to (**) in [2] and [3].

(**)* *Every indecomposable projective module contains a unique minimal submodule and is uniform.*

If further every indecomposable left projective module contains a unique minimal submodule, we call R a QF-2 ring following Thrall [7]. Hence, if R satisfies (**)*, we call R a *right* QF-2 *ring* in this note.

Let M be an R-module. If M is a homomorphic image of projective module with non-essential kernel, we call M a *non-cosmall module* [3] and [6]. Every epimorphism onto non-cosmall module has the non-essential kernel [3]. We have dealt with conditions on non-small modules in [5]. We shall consider the dual or similar conditions to them.

(*1)* *Every non-cosamll module which is contained in a projective module contains a non-zero projective summand* (dual to (*1) in [5]).
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