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ON ONE-SIDED QF-2 RINGS I

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We first consider a right artinian ring. Then every projective module P is a direct sum of indecomposable submodules; $P=\sum_I \oplus P_\alpha$. Furthermore for any simple module A in $P/\mathrm{J}(P)$ there exists a direct summand P_0 of P such that $(P_0+\mathrm{J}(P))/\mathrm{J}(P)=A$, where $\mathrm{J}(P)$ is the Jacobson radical of P. It is clear that $P\neq\sum_J \oplus P_\beta$ for any proper subset J of I.

In this paper we shall study those properties on injectives E with the condition (**) in [3] and [4], e.g. QF-2 algebra [11] (see §1). If $E=\sum_I E'_\alpha$ and $E\neq\sum_J E'_\beta$ for $J\subsetneqq I$, we say $\sum_I E'_\alpha$ be an irredundant sum. We shall give structure theorems of artinian rings over which every irredundant sum of injective in E is injective and every simple module in $E/J(E)$ is lifted to an indecomposable submodule of E. We have studied perfect rings satisfying (*) (see §1) in [4]. We shall show that they satisfy the above properties and they are right artinian from these facts.

We shall extend those ideas to more general modules in [5] and study the dual properties on projectives in [6].

1. Preliminaries

Throughout we consider a ring R with identity and every module is a unitary right R-module. Let M be an R-module. We shall denote the *Jacobson radical* and an *injective envelope* of M by $J(M)$ and $E(M)$, respectively. If M is a small sumbodule in $\mathrm{E}(M)$, M is called a *small module* [7] and [9] and otherwise we call M a *non-small module* [3]. If M contains a non-zero injective submodule, M is clearly non-small. We consider the converse case, namely

(*) *Every non-small module contains a non-zero injective submodule* [4].

In [4] we have studied perfect rings with (*). We shall show that such rings are right artinian in §4. Furthermore, we shall give some weaker conditions than (*) and show that rings satisfying new conditions give us new classes of rings.