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A REMARK ON THE CARTAN MATRIX OF A CERTAIN p-BLOCK

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1. Introduction

Let G be a finite group with order divisible by a fixed prime p. In this paper a 'block' means a 'p-block'. If B is a block of G with defect group D, we denote by C_B the Cartan matrix of B. Then it holds generally that det $C_B \geq |D|$. So it is interesting to consider when the equality sign holds in the above.

If D is cyclic, we can deduce from Dade's theorem [6] that det $C_B = |D|$. If D is a dihedral 2-group, Brauer [5], (4G) showed that det $C_B = |D|$. Also, Olsson ([9], Proposition 3.2) investigated the elementary divisors of C_B of B with quaternion or semi-dihedral defect group D.

The purpose of this paper is to prove the following

Theorem. *Let B be a block of G with defect group D and C_B the Cartan matrix of B. Suppose that the centralizer in G of any element of order p of D is p-nilpotent. Then det $C_B = |D|$, so one diviscr of C_B is $|D|$ and all other elementary divisors are* 1.

The set of elementary divisors of C_B coincides with the set of the order of defect groups of p-regular (conjugate) classes of G associated with B. (For selection of sets of conjugate classes for the blocks, see Brauer [1], [2], [4], Osima [11], and Iizuka [8].) Also the greatest elementary divisor of C_B is equal to $|D|$ and all other elementary divisors are less than $|D|$. Therefore det $C_B = |D|$ implies that $|D|$ is only one elementary divisor of C_B except 1's.

Let $^{\#}Bl_d(G)$ denote the number of blocks of G with defect d, $^{\#}Cl'_d(G)$ the number of p-regular classes of G with defect d, and p^a the order of a Sylow p-subgroup of G. The following is an immediate consequence of the theorem.

Corollary. *Suppose that the centralizer in G of any element of order p of G is p-nilpotent. Then*

$$^{\#}Bl_d(G) = {}^{\#}Cl'_d(G) \qquad \textit{for any positive integer} \quad d \leq a\,.$$

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