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COMPLETIONS OF HEREDITARY NOETHERIAN PRIME RINGS

HIDETOSHI MARUBAYASHI

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Let R be a hereditary noetherian prime ring with quotient ring Q and let $A=M_1\cap\cdots\cap M_p$ be a maximal invertible ideal of R, where $M_1, \cdots, M_p$ is a cycle (cf. [2] for the definition of cycles). The main purpose of this paper is to prove the following theorem:

Theorem 1.1. (1) *The completion $\hat{R}$ of R with respect to A is a bounded hereditary noetherian prime ring with quotient ring $Q\otimes\hat{R}$. The Jacobson radical $\hat{A}$ of $\hat{R}$ is $A\hat{R}=\hat{R}A$ and $\hat{A}^p$ is a principal right and left ideal of $\hat{R}$.*

(2) *$\hat{R}$ has the following decomposition;*

$$\hat{R} = (\overbrace{e_1\hat{R}\oplus\cdots\oplus e_1\hat{R}}^{k_1})\oplus(\overbrace{e_2\hat{R}\oplus\cdots\oplus e_2\hat{R}}^{k_2})\oplus\cdots\oplus(\overbrace{e_p\hat{R}\oplus\cdots\oplus e_p\hat{R}}^{k_p})$$

such that each $e_i\hat{R}$ is a uniform right ideal of $\hat{R}$, e_i is an idempotent in $\hat{R}$ and $e_i\hat{R}/e_i\hat{A}$ is a simple right R-module which is annihilated by M_i, where k_i is the Goldie dimension of R/M_i.

In case R is a Dedekind prime ring and A is a maximal ideal of R, Gwynne and Robson proved that $\hat{R}$ is also a Dedekind prime ring [5] (in fact, it is a principal ideal ring). We can not use their techniques to prove the theorem. The theorem is proved by using properties of cotosion R-modules.

Applying the theorem to module theory, we prove, in section 2, the following theorems:

Theorem 2.1. *Any module over $\hat{R}$ has a basic submodule.*

Theorem 2.2. *Under the same notations as in Theorem* 1.1, *any indecomposable right $\hat{R}$-module is isomorphic to one of the following $\hat{R}$-modules;*

$$e_i\hat{R}/e_i\hat{A}^n\ (n=1,2,\cdots)\,,\quad e_i\hat{R}\,,\quad e_i(Q\otimes\hat{R})\,,\quad E(e_i\hat{R}/e_i\hat{A})\ (i=1,\cdots,p)$$

where $E(e_i\hat{R}/e_i\hat{A})$ is the R-injective hull of $e_i\hat{R}/e_i\hat{A}$.