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MODULES OVER DEDEKIND PRIME RINGS. V

HIDETOSHI MARUBAYASHI

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Let R be a Dedekind prime ring, let F be a non-trivial right additive topology on R and let F_l be the left additive topology corresponding to F (cf. [8]). For any positive integer n, let F^n be the set of all right ideals containing a finite intersection of elements in F, each of which has at most n as the length of composition series of its factor module. An exact sequence $0\rightarrow L\rightarrow M\rightarrow N\rightarrow 0$ of right R-modules is EF^n-pure if the induced sequence $0\rightarrow \text{Ext}(Q_{F^n}/R, L)\rightarrow \text{Ext}(Q_{F^n}/R, M)\rightarrow \text{Ext}(Q_{F^n}/R, N)\rightarrow 0$ is splitting exact, where $Q_{F^n}=\varinjlim I^{-1}$ (I ranges over all elements in F^n). If R is the ring of integers, p is a prime number and F is the topology of all powers of p, then EF^n-purity is equivalent to p^n-purity in the sense of [12].

The aim of this paper is to investigate the structure of EF^n-pure injective modules. In Section 1, a notion of maximal F^n-torsion modules will be introduced. It is shown, in Theorem 1.10, that there is a duality between all maximal F^n-torsion modules and all direct summands of direct products of copies of $\hat{R}_{F_l^n}$ by using the results in [9], where $\hat{R}_{F_l^n}=\varprojlim R/J$ ($J\in F_l^n$). In Section 2, we shall study the category $C(F^n)$ of F^n-reduced, EF^n-pure injective modules. After discussing some properties of EF^n-puries and F^n-purities we shall give, in Theorem 2.9, characterizations of projective objects in the category $C(F^n)$. In particular, it is established that a module is a direct summand of a direct product of copies of $\hat{R}_{F_l^n}$ if and only if it is a projective object in $C(F^n)$. F is bounded if each element of F contains a non-zero ideal of R. If F is bounded, then $\hat{R}_{F_l^n}=\Pi R/P^n$, where P ranges over all prime ideals contained in F. So our results may essentially be interesting in case F contains completely faithful right ideals of R in the sense of [3].

1. The Harrison duality

Throughout this paper, R will be a Dedekind prime ring with the two-sided quotient ring Q and $K=Q/R\neq 0$. By a module we shall understand a unitary right R-module. In place of $\otimes_R$, Hom_R, Ext_R and Tor^R, we shall just write $\otimes$, Hom, Ext and Tor, respectively. Since R is hereditary, $\text{Tor}_n=0=\text{Ext}^n$ for all $n>1$ and so we shall use Ext for Ext^1 and Tor for Tor_1. Let I be