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ON A CHARACTERIZATION OF SURFACES CONTAINING CYLINDERLIKE OPEN SETS

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Introduction. Let k be an algebraically closed field of characteristic zero. Let V be a nonsingular projective surface defined over k and let D be a reduced effective divisor on V. Consider the following four conditions:

(1) There exists a nonempty open set U in V-Supp (D) such that U has a structure of trivial $\boldsymbol{A}^1$-bundle; U is called a cylinderlike open set;

(2) There exists an irreducible curve C on V such that $C \not\subset \mathrm{Supp}\,(D)$ and $(C \cdot D+K)<0$, where K is the canonical divisor on V;

(3) for any divisor A on V, $|A+m(D+K)|=\phi$ for all sufficiently large integer m;

(4) $|m(D+K)|=\phi$ for every positive integer m.

If D satisfies the condition that V-Supp (D) is affine and Supp (D) has only normal crossings as singularities, then the above four conditions are equivalent to each other. In effect, the equivalence of the first three conditions and the implication (3)$\Rightarrow$(4) are proved in the previous paper with Miyanishi [MS]. The implication (4)$\Rightarrow$(3) was proved by Fujita [F].

In the first part of this paper, we shall prove the following

Theorem. *With the notations as above, assume that the following conditions are satisfied*:

(i) V-Supp (D) *contains no excpetional curve of the first kind and* Supp (D) *is connected*;

(ii) Supp (D) *has only normal crossings as singularities*;

(iii) *write* $D=\sum_{i=1}^{r} C_i$, *where* C_i *is an irreducible component*; *then the* $(r\times r)$*-matrix* $((C_i \cdot C_j)_{1\leq i,j\leq r})$, *which we call simply the intersection matrix of* D, *is not negative definite. Then the above four conditions are equivalent to each other.*

This theorem does not hold if one drops off the condition that Supp (D) is connected. In the second part, we shall show this by constructing a counter-example.

We retain in this article the terminology and notations of the previous